quantity is given by the expression \( m_\omega (1 - \nu_\omega) \gamma^* \).

The value of the permeability coefficient has almost no effect on the position of the fusion-sublimation front, but determines the position of the condensation front. The value of \( \gamma^* \) increases monotonically with \( k \). For small \( k \) the vapor-water mixture zone occupies almost the entire region free of ice. For example, when \( k = 10^{-18} \text{ m}^2 \) we get \( P^* = 1710.5 \text{ Pa}, T^* = 288.3^\circ \text{K}, \gamma^* = 0.034, \gamma_R = 0.36 \). Obviously, in this case the thawed soil has absorbed less than 10% of the possible volume of water. As \( k \) tends to the critical value, the width of the vapor interlayer tends to zero. For example, when \( k = 0.95 \times 10^{-18} \text{ m}^2 \) we have \( P^* = 647.1 \text{ Pa}, T^* = 274.2^\circ \text{K}, \gamma^* = 0.336, \gamma_R = 0.358 \). Accordingly, the pressure and temperature on the condensation surface tend to the values at the triple point.

LITERATURE CITED

STABILITY OF VISCOUS FLOW BETWEEN ROTATING AND STATIONARY DISKS

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The linear problem of the stability of viscous flow between rotating and stationary parallel disks is solved in the locally homogeneous formulation using the method of normal modes. The main flow is assumed to be self-similar with respect to the radial coordinate. The system of sixth-order equations, derived for the amplitude functions of the disturbances, is integrated by a finite difference method. The stability characteristics with respect to disturbances of four types are calculated.

The experiments [1] indicate a variety of secondary flow regimes between a rotating and a stationary disk. In [2] for formulating the linear stability problem it was proposed to use the locally homogeneous approximation, which had been successfully employed on previous equations for analyzing the stability of the boundary layer on an exposed rotating disk [3-5]. In [6] in solving the stability problem it was assumed that the main flow between the disks is self-similar along the radius.

Below, we present the results of calculations that supplement and refine those made in [6]. For the first time the calculated stability limits and critical perturbation parameters are compared in detail with the experimental data.

1. Main Flow

Let a viscous incompressible fluid occupy the gap between infinite parallel disks, one of which (the lower) rotates at a constant angular velocity \( \omega \), while the other is stationary. We denote the distance between the disks by \( s \). The origin of the cylindrical coordinate system \( r, \theta, z \) is located at the center of the plane of the rotating disk. The main steady flow possesses the properties of axial symmetry and self-similarity with respect to the radial coordinate [7]:

\[
\begin{align*}
V_r &= \omega r \varphi (\eta), \\
V_\theta &= \omega r \varphi (\eta), \\
V_z &= -2 \alpha_s s \varphi (\eta), \\
\frac{P}{\rho} &= \omega^2 s^2 \varphi (\eta) + \frac{1}{2} \Lambda \omega^2 r^2, \\
\eta &= \frac{z_s}{s}
\end{align*}
\] (1.1)

where: $\Lambda = \text{const}$, and $P$ is the pressure. Substituting (1.1) in the Navier–Stokes equation leads to the following system of ordinary differential equations:

$$g=f', \quad g''/\gamma+2fg'-g^2=\Lambda-\varphi^2, \quad q''/\gamma+2f'q'-2gq=0, \quad \Pi''=-2g'/\gamma-2gf$$

(1.2)

Here, $\gamma = \omega s^2/v$ is the rotation parameter, and a prime denotes differentiation with respect to $\eta$. The boundary conditions for (1.2) have the form:

$$j=g=0, \quad (q=1, \eta=0), \quad (q=0, \eta=1)$$

(1.3)

Numerical solutions of the problem (1.2), (1.3) were found by the stabilization method; the approximation of the derivatives with respect to $\eta$ by finite differences had second-order accuracy.

When $\gamma > 220$ the solution of problem (1.2), (1.3) is not unique [8]: for $\gamma = 625$ 20 solutions have been obtained [9]. However, only the solutions of Batchelor [7] and Stewartson [10] are of practical interest. The first of these, namely, continuous continuation from the domain $\gamma < 220$, describes a flow with a rotating core and two wall boundary layers; for very large $\gamma$ the core rotates as a rigid body with an angular velocity equal to $0.313\omega$. In the Stewartson solution the velocities are significant only in a thin layer on the rotating disk.

Problems of the stability of the main flow described by Stewartson’s solution in fact reduce to problems for the layer on an exposed rotating disk. Consequently, on the interval $\gamma > 220$ we solved our stability problem for the main flow described by Batchelor’s solution.

2. Linear Stability Problem

Bearing in mind the results of the experiments carried out in [1] and following [3], we introduce the curvilinear orthogonal fixed coordinate system $x, y, z$, whose origin is located in the plane of the rotating disk on a circle of radius $r_0$; the coordinate lines $x_1$ and $z_1$ lie along logarithmic spirals, and the angle $\varepsilon$ between the direction of the radius and the coordinate line $x_1$ is positive counterclockwise. The relation between $x_1, y_1, z_1$ and the cylindrical coordinates $r, \theta, z$, the Lamé coefficients and their derivatives with respect to the coordinates are given by the expressions

$$x=r_0[\ln(r/r_0)\cos\varepsilon+(0-\varepsilon_0)\sin\varepsilon], \quad y=y_1, \quad z=z_1,$$

(2.1)

$$H_0=\frac{\partial H_0}{\partial x}, \quad \frac{\partial H_0}{\partial z}, \quad \sin\varepsilon, \quad \frac{\partial^2 H_0}{\partial z^2}, \quad \frac{\partial^2 H_0}{\partial x^2}, \quad \frac{\partial^2 H_0}{\partial z^2}, \quad \frac{\cos\varepsilon}{r_0^2}, \quad \frac{\cos\varepsilon}{r_0^2}, \quad \frac{\sin\varepsilon}{r_0^2} (r=r_0)$$

The infinitesimal perturbations of the velocity $u$ and pressure $p$ in the neighborhood of $r = r_0$ are assumed to be independent of the $z_1$ coordinate and periodic with respect to $x_1$ and time $t$:

$$u=(u_1(y_1), u_2(y_1), u_3(y_1))\exp(i(\alpha x_1+\beta t)), \quad p=p_1(y_1)\exp[i(\alpha x_1+\beta t)]$$

(2.2)

We derive the amplitude equations in the following order. We introduce dimensionless variables and write the equations for the main flow:

$$x=x_1, \quad y=y_1, \quad z=z_1, \quad U=U_1, \quad V=V_1, \quad W=W_1,$$

(2.3)

$$U=r_0U_1, \quad U_1=g(y)\cos\varepsilon+q(y)\sin\varepsilon, \quad V=-2f(y), \quad W=rW_1, \quad W_1=g(y)\sin\varepsilon-q(y)\cos\varepsilon$$

We substitute relations (2.2), (2.3) in the linearized Navier–Stokes equations written for $u_2, v_2, w_2$, and $p_2$ in the system $x_1, y_1, z_1$, and using (2.1) estimate the individual terms on the assumption that the quantities $1/Re = \sqrt{\omega r_0 s}$ and $k_1=s/r_0=\gamma/Re$ are small.