CONSTRUCTION OF A SOLUTION OF A QUASILINEAR PARTIAL DIFFERENTIAL EQUATION OF PARABOLIC TYPE WITH OSCILLATING AND SLOWLY VARYING COEFFICIENTS

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We study a boundary-value problem for a partial differential equation of parabolic type with coefficients in the form of Fourier series with coefficients and frequency slowly varying in time.

In recent years, there has been extensive development of the theory of partial differential equations with periodic coefficients [1, 2]. In this paper, we extend the results obtained in [3, 4] to the partial differential equations of parabolic type.

Consider the boundary-value problem

\[ \frac{\partial u}{\partial t} = a(t, \varepsilon) \frac{\partial^2 u}{\partial x^2} + b(t, \varepsilon)u + c(x, t, \varepsilon, \theta(t, \varepsilon)) + \mu f(x, t, \varepsilon, \theta(t, \varepsilon), u, \frac{\partial u}{\partial x}), \]

\[ 0 \leq x \leq l < +\infty, \quad -\infty < t < +\infty, \quad 0 \leq \varepsilon \leq \varepsilon_0 < +\infty, \]

\[ u(0, t, \varepsilon) = p(t, \varepsilon, \theta(t, \varepsilon)) \equiv \sum_{n=-\infty}^{\infty} p_n(t, \varepsilon)e^{in\theta(t, \varepsilon)}, \]

\[ u(l, t, \varepsilon) = q(t, \varepsilon, \theta(t, \varepsilon)) \equiv \sum_{n=-\infty}^{\infty} q_n(t, \varepsilon)e^{in\theta(t, \varepsilon)}, \]

\[ c(x, t, \varepsilon, \theta(t, \varepsilon)) = \sum_{n=-\infty}^{\infty} c_n(x, t, \varepsilon)e^{in\theta(t, \varepsilon)}, \]

\[ \theta(t, \varepsilon) = \int_{0}^{t} \varphi(t, \varepsilon)d\tau, \quad 0 < \varphi_0 \leq |\varphi(t, \varepsilon)| \leq \varphi^0 < +\infty, \]

\[ \frac{d\varphi(t, \varepsilon)}{dt} = \varepsilon\varphi(t, \varepsilon) + \sum_{n=-\infty}^{\infty} \sup_{t, \varepsilon} \left[ \left| p_n(t, \varepsilon) \right| \right] < +\infty, \]

\[ \sup_{t, \varepsilon} \left[ \left| a(t, \varepsilon) \right| \right] < +\infty, \quad \sum_{n=-\infty}^{\infty} \sup_{x, t, \varepsilon} \left| c_n(x, t, \varepsilon) \right| < +\infty, \]

\[ \frac{d}{dt} \left\{ \begin{array}{l} p_n(t, \varepsilon) \\ q_n(t, \varepsilon) \end{array} \right\} = \varepsilon \left\{ \begin{array}{l} \tilde{p}_n(t, \varepsilon) \\ \tilde{q}_n(t, \varepsilon) \end{array} \right\}, \quad \sup_{n, t, \varepsilon} \left| \tilde{p}_n(t, \varepsilon) \right| < +\infty, \]

\[ a(t, \varepsilon), b(t, \varepsilon), c(x, t, \varepsilon, \theta(t, \varepsilon)) \]

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\[ \frac{d}{dt}\left\{ a(t, \varepsilon) \right\} = \varepsilon \left\{ \bar{a}(t, \varepsilon) \right\}, \quad \sup_{t, \varepsilon} \left\{ |\bar{a}(t, \varepsilon)| \right\} < +\infty, \]

\[ \frac{\partial c_n(x, t, \varepsilon)}{\partial t} = \varepsilon \bar{c}_n(x, t, \varepsilon), \quad \sup_{n, x, t, \varepsilon} |\bar{c}_n(x, t, \varepsilon)| < +\infty, \tag{3} \]

where \( \mu \) is a real positive parameter.

We impose the following restriction on the function \( f \): Let

\[ u = \sum_{n=-\infty}^{\infty} u_n(x, t, \varepsilon)e^{in\Theta(t, \varepsilon)}. \tag{4} \]

Then

\[ f = \sum_{n=-\infty}^{\infty} f_n(x, t, \varepsilon, U, \frac{\partial U}{\partial x})e^{in\Theta(t, \varepsilon)}, \tag{5} \]

where \( U = \text{col} (u_n), \ n \in \mathbb{Z}. \)

We study whether problem (1)-(3) has a solution of the form (4) with \( u_n(x, t, \varepsilon) \) possessing the properties

\[ \sum_{n=-\infty}^{\infty} \sup_{x, t, \varepsilon} |u_n(x, t, \varepsilon)| < +\infty, \]

\[ \frac{\partial u_n(x, t, \varepsilon)}{\partial t} = \varepsilon \bar{u}_n(x, t, \varepsilon), \quad \sup_{n, x, t, \varepsilon} |\bar{u}_n(x, t, \varepsilon)| < +\infty. \]

By the substitution

\[ u = p + \frac{x}{l} (q - p) + v, \]

where \( v \) is a new unknown function, we reduce problem (1)-(3) to a problem with homogeneous boundary conditions

\[ \frac{\partial v}{\partial t} = a(t, \varepsilon)\frac{\partial^2 v}{\partial x^2} + b(t, \varepsilon)v + c^*(x, t, \varepsilon, \Theta(t, \varepsilon)) + \mu f^*(x, t, \varepsilon, \Theta(t, \varepsilon), v, \frac{\partial v}{\partial x}), \tag{6} \]

\[ v(0, t, \varepsilon, \Theta) = v(l, t, \varepsilon, \Theta) = 0. \tag{7} \]

We seek solutions of problem (6), (7) in the form

\[ v(x, t, \varepsilon, \Theta(t, \varepsilon)) = \sum_{n=-\infty}^{\infty} v_n(x, t, \varepsilon)e^{in\Theta(t, \varepsilon)}. \tag{8} \]

Substituting (8) into equality (6) and taking conditions (7) into account, we obtain