Proof. From the infinitesimal analog of the G-invariance of the element t (see the proof of Proposition 2), it is clear that \([t, \exp (X \otimes 1 + 1 \otimes X)] = 0 \forall X \in \mathfrak{g} \). Since \(\exp (X \otimes 1 + 1 \otimes X) = \exp (X) \otimes \exp (X)\), the element t commutes with the element \(g \otimes g\) for every \(g \in G\). Therefore, \([\rho(t), \rho(g) \otimes \rho(g)] = 0\). Setting \(\rho(g) = (\rho(S) - c_1 1)/c_2\) and substituting in the latter equality, we obtain, following obvious algebraic transformations using the G-invariance properties of the element t, that \([\rho(t), \rho(S) \otimes \rho(S)] = 0\).

COROLLARY. If on every orbit in M, the following identity is satisfied:

\[a_0 (\rho(S))^2 + a_1 \rho(S) + a_2 = 0,\]

where \(a_0, a_1, a_2 \in \mathbb{C}\) are unique for each orbit, the mapping \(\rho\) satisfies condition (*).

Proof. We first expand the expression \(\exp (\rho(S))\) in a series and apply the given identity after expressing all the terms of degree higher than the first in the form of a linear combination of the matrices I and \(\rho(S)\). The theorem may then be applied.

REFERENCES


SIMILITUDE OPERATORS GENERATED BY NONLOCAL PROBLEMS FOR SECOND-ORDER ELLIPTIC EQUATIONS

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The spectral properties and properties of the \(L^2\)-solutions of the nonlocal problem for second-order linear elliptic nondivergent-type equations that represent an isospectral disturbance of the Dirichlet problem are investigated.

Nonlocal elliptic problems have been investigated in [1—7]. In the course of these studies relations were established between the operator of the nonlocal (perturbed) problem and the structurally simplest equations, i.e., the boundary conditions of the boundary-value (undisturbed) problem. In [6] an isospectral relation was understood as just such a relation. In [7] nonlocal problems for integro-differential equations were investigated by means of the method of similitude operators. In the present study the spectral properties and solvability conditions of the solutions of the nonlocal problem are studied on the basis of an analog of the method of similitude operators, itself based on the symmetry properties of the domain of the equation and the boundary conditions of [8].

Suppose that $Q$ is a bounded domain in $\mathbb{R}^{n-1}$ with boundary $\Gamma \in \mathcal{C}^\infty$, $\Omega = (0, 1) \times Q \subset \mathbb{R}^n$, and let $(\Gamma_p)_{p=1}^q$ be an ordered sequence of manifolds in $\Omega$ diffeomorphic to $Q$; $a_{ij}(x), a(x) \in \mathcal{C}^\infty(\Omega), a_{ij} = a_{ji} \in \mathbb{R}$, $a \in \mathbb{R}$, $i, j = 1, \ldots, n$. Let us assume that the surface $\Gamma_p$ is determined by the equations $x_1 = \varphi_p(x')$, $x = (x_1, x')$, $x_1 \in (0, 1)$, $x' \in Q$, and we specify that

$$b_p(x) \in \mathcal{C}^\infty(\Gamma_p), \quad b_p(x) = 0, \quad x \in \partial\Omega \cap \Gamma_p, \quad p = 1, q.$$

By $H^s(\Omega)$ we denote the Sobolev space $(2s \in \mathbb{Z}_+, s \geq 0)$. In $H^2(\Omega)$ a norm induced by the following scalar product is specified:

$$(u, v)_{H^s(\Omega)} = (u, v)_{L^2(\Omega)} + (\Delta u, \Delta v)_{L^2(\Omega)}, \quad \Delta = \sum_{i=1}^n \frac{d^2}{dx_i^2}.$$

Let $B: H^2(\Omega) \rightarrow H^{3/2}(Q)$ be an operator of the form

$$Bu = \sum_{i=1}^q b_i(\varphi_i(x'), x') u(\varphi_i(x'), x'), \quad x' \in \overline{Q}, \quad u \in H^2(\Omega),$$

and

$$W = L^2(\Omega) \oplus H^{3/2}(S) \oplus H^{3/2}(Q) \oplus H^{3/2}(Q)$$

be a Hilbert space with norm

$$||F||_W = ||f||_{L^2(\Omega)} + ||\varphi||_{L^2(\Omega)} + ||\varphi_0||_{H^{3/2}(Q)} + ||\varphi_1||_{H^{3/2}(Q)},$$

where $F = (f, \varphi, \varphi_0, \varphi_1) \in W$, $S = (0, 1) \times \partial Q$.

In the domain $\Omega$ we wish to consider the nonlocal problem

$$Lu = -\sum_{i,j=1}^n a_{ij} \frac{\partial^2}{\partial x_i \partial x_j} u(x) + a(x) u(x) = f(x), \quad x \in \Omega,$$

$$iu = u |_{x \in S} = \varphi,$$  \hspace{1cm} (1)

$$l_{Bu} = u |_{x = i} + Bu = \varphi_1, \quad x' \in \overline{Q}, \quad i = 0, 1,$$  \hspace{1cm} (2)

$$Bu = \sum_{i=1}^q b_i(x) u(x) |_{x \in \Gamma_i} \quad \text{d.f.}$$  \hspace{1cm} (3)

By a solution of problem (1) — (3) we will understand a function $u \in H^2(\Omega)$ that satisfies Eq. (1) in the sense of equality of functions in $L^2(\Omega)$ and conditions (2) and (3) in the sense of equality of the functions $H^{3/2}(S)$ and $H^{3/2}(Q)$, respectively.

By $L_B$ we denote the operator of problem (1) — (3), $L_B u \overset{\text{d.f.}}{=} Lu$, $u \in D(L) = \{ u \in H^2(R) : lu = 0, \quad l_{Bu} = l_{Bu} = 0 \}$.

The number $\lambda \in \mathbb{C}$ such that there exists a nontrivial solution of the equation $Lu = \lambda u$ from $D(L_B)$ and such that the solution itself is an eigenfunction will be called the eigenvalue of $L_B$.

By $\sigma(L_B)$ we denote the set of eigenvalues enumerated in nondecreasing order of absolute values $|\lambda_m|$, and by $\nu(L_B)$ the system of eigenfunctions of $L_B$.

It is easily seen that whenever $b_j = 0$, $j = 1, q$, that is, $B = 0$, the homogeneous conditions (2) and (3) constitute a Dirichlet condition:

$$u |_{x \in \partial Q} = \varphi.$$  \hspace{1cm} (4)