A theorem for representation of the solution of a nonhomogeneous linear differential equation with constant coefficients in $D'$ in the form of a convolution of the right side and the fundamental function is generalized to the case of a linear nonhomogeneous differential equation with infinitely differentiable coefficients. Based on this theorem, a method is proposed for investigating both direct and inverse boundary-value problems in a distribution space.

Let $L(x, D) = \sum_{|\alpha| \leq \omega} a_\alpha(x) D^\alpha$ be a hyper-elliptic differential operator with coefficients $a_\alpha(x) \in C^\infty (\mathbb{R}^n)$, $\xi(x, y)$ a normal fundamental function such that the solution in $D'(\mathbb{R}^n)$ of the equation $L(x, D) \xi(x, y) = \delta(x - y)$, where $L^\omega(y, D) \xi(x, y) = \delta(x - y)$. Suppose that $Z(\mathbb{R}^n) = \{ \psi(y) = (\xi(x), \xi(x, y)), \psi \in D(\mathbb{R}^n) \} (Z \subset C^\infty$ for the hyper-elliptic operator $L)$, $D \subset V \subset C^\infty$. We denote by $D' \cap V$, a space of generalized functions $f(x, y)$ in $D'(\mathbb{R}^n \times \mathbb{R}^n)$, which belong to $V$ as functions of $y$, that is, $[1](\psi(x), f(x, y)) \in V(\mathbb{R}^n)$ for arbitrary $\psi \in D(\mathbb{R}^n)$ and $(\psi_n(x), f(x, y)) \to 0$, whenever $\psi_n(x) \to 0$. It is clear that in the case of a hyper-elliptic operator $L \xi(x, y) \in D' \cap C^\infty$ and $L^* (y, D) (\psi(x), \xi(x, y)) = (\psi(x), L^* (y, D) \xi(x, y)) = (\psi(x), \delta(x - y) = \psi(y)$, $L^* (y, D) \psi(x), \xi(x, y)) = (\psi(x), L (x, D) \xi(x, y)) = (\psi(y), \Phi(y) \in C^\infty (\mathbb{R}^n)$.

We will refer to the functional $f_p(x) = f(x, y) \otimes F(y)$ as the composition of $f(x, y) \in D' \cap V$, and $F(y) \in V'(\mathbb{R}^n)$, where the functional is determined by the equality $(\psi(x), f_p(x)) = (\Phi(x), f(x, y) \otimes F(y))$ for every $\psi \in D(\mathbb{R}^n)$. Linearity and continuity of the composition is easily proved, so that $f_p(x) \in D'(\mathbb{R}^n)$.

THEOREM 1. Let $F \in Z'(\mathbb{R}^n) (F \in D'(\mathbb{R}^n))$. The generalized function $u(x) = \xi(x, y) \otimes F(y)$ is the solution in $Z'(\mathbb{R}^n) (D'(\mathbb{R}^n))$ of the equality

$$L(x, D) u = F$$

and is unique in the class of distributions in $D'(\mathbb{R}^n)$ for which the composition of a normal fundamental function is defined.

In fact, for every $\psi \in Z(\mathbb{R}^n) (\psi \in D(\mathbb{R}^n))$

$$(\psi, Lu) = (L^* \psi, u) = (L^* \psi(x), \xi(x, y) \otimes F(y)) = ((L^* \psi(x), \xi(x, y)), F(y)) = (\psi(y), F(y))$$

$$(\psi, Lu) = (L^* \psi(x), \xi(x, y)) = (L^* \psi(x), F(y)) = (\psi(y), F(y))$$

If $u_1(x)$ and $u_2(x)$ are two solutions of an equation from $D'(\mathbb{R}^n)$, then for $u(x) = u_1(x) - u_2(x)$ we will have that $L(x, D)u(x) = 0$ in $D'(\mathbb{R}^n)$, and for every $\psi \in D$, $\psi(u) = ((L^* \psi(x), \xi(x, y)), u(y)) = (L^* (y, D) \psi(x), \xi(x, y)) \psi(y), F(y))$.

In [2-13] elliptic, parabolic, and hyperbolic boundary-value problems in different spaces of generalized functions were studied from different approaches. In [2, 4, 7, 9-13] representations of solutions of auxiliary Green's functions were found, and the solutions of boundary-value problems that had been obtained in [12, 13] were reduced in certain other works to the solution of integral equations of the second kind in spaces of smooth functions. A method of solving direct and inverse boundary-value problems in spaces of distributions based on the use of Theorem 1 was also investigated, restricted to the case of a second-order parabolic operator.

Let \( \Omega_0 \) be a region in \( \mathbb{R}^n \) bounded by a closed \((n - 1)\)-dimensional surface \( \Omega_1 \) of class \( C^\infty \), \( \Omega_1 = \Omega_1 \times (0; T) \), \( i = 0, 1 \); with \( u(x, t) \) the solution of the problem

\[
Lu(x, t) = F_0(x, t), \quad (x, t) \in \Omega_0,
\]

\[
u(x, t) = F_1(x, t), \quad (x, t) \in \Omega_1,
\]

\[
u(x, 0) = F_2(x), \quad x \in \Omega_0
\]

\[
Lu(x, t) = u_t - \sum_{i,j=1}^n a_{ij}(x, t) u_{x_i x_j} + \sum_{i=1}^n a_i(x, t) u_{x_i} + a_0(x, t) u
\]

(2)

for sufficiently smooth \( F_0, F_1, \) and \( F_2 \),

\[
\tilde{u}(x, t) = \begin{cases} u(x, t), & (x, t) \in \Omega_0; \\ 0, & (x, t) \notin \Omega_0. \end{cases}
\]

Then \( \tilde{u}(x, t) \) is a regular generalized function and for every \( \psi(x, t) \in D(\mathbb{R}^{n+1}) \),

\[
(\psi, L\tilde{u}) = (L^* \psi, u) = \int_{\Omega_0} L^* \psi u dxdt = \int_{\Omega_0} (\psi F_0 dxdt + \int_{\Omega_1} (\psi Bu - C\psi F_1) dxdt +
\]

\[
+ \int_{\Omega_0} \{\psi(x, 0) F_2(x) - \psi(x, T) u(x, T)\} dx,
\]

(3)

where

\[Bu = a \frac{du}{dN} + \beta u, \quad Cu = a \frac{du}{dN} + (\beta - b) u, \quad a = \left[\sum_{k=1}^n \left(\sum_{h=1}^n a_{kh} n_h\right)^2\right]^{1/2},\]

\[b = \sum_{h=1}^n e_h n_h, \quad e_i = a_0 - \sum_{k=1}^n \frac{\partial a_{ih}}{\partial x_k}, \quad \beta \in C^\infty(\overline{\Omega_1}), \quad n_i = n_i(x)\]

are the directional cosines of the external normal \( n(x), \), \( \upsilon(x) \) is its unit vector, \( N_i = N_i(x, t) = \frac{1}{a(x, t)} \sum_{h=1}^n a_{ih}(x, t) n_h(x) \) are the directional cosines of the conormal, and \( d/DN \) is the differentiation operator on the conormal.

Let us now introduce generalized functions \( F_{Q_0}, C^\infty F_1 \) in \( D'(\mathbb{R}^{n+1}) \) with compact support on \( \Omega_1 \), such that \( (\psi, F_{Q_0}) = (\psi, F_0) \), \( (\psi, C^\infty F_1) = (C\psi, F_1) \) for every \( \psi \in C^\infty(\mathbb{R}^{n+1}) \), where \( \langle \psi, F \rangle \) represents the action of the generalized function \( F \in D'(\Omega_1) \) on the fundamental function \( \psi \in D(\Omega_1) = C^\infty(\overline{\Omega_1}) \). Moreover, \( \langle \psi, F_{Q_0} \rangle = \int_{\Omega_0} \{\psi(x, 0) F_2(x)\} dx \) for regular \( F_{Q_1} \) and \( F_1 \). Let us also introduce a generalized function \( F_{2T} \in D'(\mathbb{R}^{n+1}) \) with support on \( \Omega_0 \) such that for every \( \psi \in C^\infty(\mathbb{R}^{n+1}) \) \( (\psi, F_{2T}) = (\psi(x, 0), F_{2T}) \), where \( \langle \psi, F \rangle \) represents the action of \( F \in D'(\Omega_0) \) on \( \psi \in D(\Omega_0) = C^\infty(\overline{\Omega_0}) \).

Now (3) may be written in the form

\[
(\psi, L\tilde{u}) = (\psi, \tilde{F}_0 + F_0 - C^\infty F_1 + F_2 - F_{2T}), \quad \psi \in D, \quad F_{Q_0} = B\tilde{u}\big|_{\partial \Omega_1}, \quad F_{2T} = \tilde{u}\big|_{\partial \Omega_1}.
\]

That is, \( \tilde{u} \) is a solution belonging to \( D'(\mathbb{R}^{n+1}) \) of the equation

\[
L\tilde{u} = \tilde{F}_0 - F_{Q_0} - C^\infty F_1 + F_2 - F_{2T}
\]

(4)

587