The accuracy of forecasts of the screening frequency of the middle-latitude sporadic-E layer is analyzed. It is shown that the accuracy of screening-frequency forecasts is a function of time of day and the a priori screening frequencies used in the forecast.

The screening frequency $f_b(t)$ is of great practical importance in forecasting of the conditions for ionospheric VHF radio communication with allowance for the E_s layer, since it is most closely related to the maximum plasma frequency of the E_s layer [1]. The accuracy of $f_b(t)$ forecasting can be a complex function of the time of day, the lead time of the forecast, a priori knowledge, etc., since $f_b(t)$ represents a non-Gaussian nonstationary process [2]. Analysis of these dependences is necessary for the practical use of $f_b(t)$ forecasts.

A statistical model of the screening frequency of the middle-latitude E_s layer obtained earlier [3] using wind-shear theory [4] has the form

$$ z_t = (\beta_1 + \beta_2)z_{t-1} + \beta_1\beta_2z_{t-2} = \xi_t, $$

where $z_t = \ln[f_b(t)] - m(t)$, $\xi_t$ is white noise with a zero mean value and variance $\sigma^2$, readings $z_t$ and $z_{t-1}$ are separated by 15 min, and $m(t)$ is a determined periodic trend with a principal period of 24 h, which is approximated by two terms of a Fourier series:

$$ m(t) = a_0/2 + \sum_{i=1}^{\infty} \left[a_i \cos(2\pi it/96) + b_i \sin(2\pi it/96)\right]. $$

The model contains eight parameters, estimates of which, obtained by the maximum-likelihood method from experimental $f_b$ data from a vertical-sounding station in Moscow during June-July 1976, were $\beta_1 = 0.81$, $\beta_2 = -0.14$, $\sigma^2 = 0.07$, $a_0 = 1.75$, $a_1 = -0.42$, $a_2 = -0.1$, $b_1 = 0$, and $b_2 = -0.21$. All numerical calculations were performed for these parameter values.

A forecast with a minimum standard deviation (SD) of a process $z_t$ from time $t$ to time $t + l$, $l = 1, 2, \ldots$, i.e., with lead time $l$, will be determined, owing to the Markovian properties of process $z_t$, by only two known readings of process $z_t$ at times $t$ and $t - k$, $k = 1, 2, \ldots$ that are closest to the time at which the forecast is given:

$$ \hat{z}_{t+1} = \frac{1}{1-r^2} \left[ (r_1 - r_k r_{k+1})z_t + (r_{k+1} - r_k r_1)z_{t-k} \right], $$

where $r_1$ is the normalized autocorrelation function of process $z_t$,

$$ r_1 = \frac{(1 - \beta_2^2)\beta_1^{k+1} - (1 - \beta_1^2)\beta_2^{k+1}}{(\beta_1 - \beta_2)(1 + \beta_1 \beta_2)}. $$

The dispersion of forecast (3) has the form

$$ d_z(1) = \sigma^2 \left[ 1 - \frac{r_1^2 + r_{1-k}^2 - 2r_1 r_{1-k}}{1 - r^2} \right]. $$
where $\sigma_z^2 = \sigma^2/[1 - (\beta_1 + \beta_2)\rho_1 + \beta_1\beta_2\rho_2]$ is the variance of process $z_t$.

From the forecast (3) of process $z_t$ we move to a forecast of the screening frequency. An $f_b(t)$ forecast with lead time $l$ with minimal SD has the form

$$ f_{b, l}^*(t) = \exp\{z_t(l) + m(t+1) + d_z(l)/2\}. $$

The dispersion of forecast (4) is a function of $l$, $k$, $t$, $z_t$, and $z_{t-k}$:

$$ D(l, t) = \exp\left\{2\left[z_t(l) + m(t+1)\right] + d_z(l)\right\}\left[\exp\left\{d_z(l)\right\} - 1\right]. $$

To simplify the analysis of formula (5), we convert to the dimensionless dispersion $D_0(l, t)$ by averaging (5) over $z_t$ and $z_{t-k}$:

$$ D_0(l, t) = \exp\left\{2\sigma_z^2(\mu_{1,k}^2 + \nu_{1,k}^2 + 2\mu_{1,k}\nu_{1,k}\rho_{1,k}) + 2m(t+1) + d_z(l)\right\}\left[\exp\left\{d_z(l)\right\} - 1\right], $$

where $\mu_{1,k} = (\rho_1 - \rho_{1,k}\rho_{1,k+1})(1 - \rho_2^2)$ and $\nu_{1,k} = (\rho_{k+1} - \rho_{1,k}\rho_{1,k})(1 - \rho_2^2).$ Since $D_0(l, t)$ is a periodic function of time with a 24-h period (96 readings every 15 min), formula (6) can be averaged over the time of day:

$$ D_0(l, t) = \frac{1}{96} \sum_{i=1}^{96} D_0(l, t). $$

Analysis of formulas (5)-(7) has shown that the dispersion of an $f_b(t)$ forecast is weakly dependent on $k$ for the examined numerical values of the parameters of model (1). Subsequently, therefore, the entire analysis of formulas (5)-(7) was performed for $k = 1$. The SD of the forecast $S_0(l, t) = \sqrt{D_0(l, t)}$ is shown in Fig. 1 as a function of the lead time $l$. As $l \to \infty$, the forecast SD approaches the standard deviation of the process $f_b(t)$ averaged over the time of day.

More-detailed information on the accuracy of an $f_b(t)$ forecast is provided by the dispersion of (6). The unconditional SD of a forecast $S_0(l, t) = \sqrt{D_0(l, t)}$ is shown in Fig. 2 as a function of time of day $\tau = (t + l)0.25$ h for which the forecast is made. Curves 1-3 correspond to lead times $l = 1, 3, and 20$, respectively. It is apparent from Fig. 1 that the accuracies of screening-frequency forecasts for different times of day can differ by a factor of 3 and can amount to 1.3 MHz in absolute value. Smaller forecast errors correspond to smaller average screening frequencies and vice versa.

The dispersion of the forecast (5) is of greatest interest to the user, since it characterizes the accuracy of a specific forecast. We shall see how the dispersion (5) behaves on the average over a 24-h period. For this, we move from $D(l, t)$ (5) to the characteristic

$$ S(l) = \left[\frac{1}{96} \sum_{i=1}^{96} D(l, t)\right]^{1/2} $$