An analytical expression for the two-frequency correlation function of reflected radiation \( p = \langle \hat{U}(\omega_1) \hat{U}(\omega_2) \rangle \) is derived in the framework of the Kirchhoff approximation, assuming that the mean square roughness heights \( \sigma_{1,2} \) of the upper (\( \sigma_1 \)) and lower (\( \sigma_2 \)) boundaries are large compared to the wavelength \( \lambda \) and taking account of large-scale permittivity fluctuations \( \delta \varepsilon \) in the layer. The condition under which \( p \) cannot be small when \( \sigma_2^2 \gg \lambda^2 \) is specified. In particular, it is shown that if the scattering is only at the upper boundary of the layer (when \( \sigma_1 \neq 0, \sigma_2 = \delta \varepsilon = 0 \)), then this condition is \( \omega_1 (1 - m \sqrt{\varepsilon}) = -\omega_2 (1 - n \sqrt{\varepsilon}) \), where \( m, n = 0, 1, \ldots \). The potential of the layered medium sounding methods based on the relations obtained is estimated.

The scattering of radio waves by a two-layer medium with statistically rough boundaries, where the inhomogeneities are large in scale but small compared to the wavelength \( \lambda \), was considered in [1]. There, the authors obtained and investigated analytical expressions for the coherent \( P_c \) and noncoherent \( P_{nc} \) components of the reflected power for an arbitrary correlation between the inhomogeneities of the boundaries. In the present paper, the authors consider the case of large-scale inhomogeneities of the boundaries, taking account of permittivity fluctuations in the layer. The correlation between the inhomogeneities of the boundaries and between the inhomogeneities of the boundaries and of the layer is assumed to be constant.

Consider a two-layer medium where \( H \) is the mean thickness of the layer, \( h_{1,2}(x, y) \) are the vertical (along the \( z \) axis) deviations of the upper (1) and lower (2) boundaries from their average position, \( \varepsilon_2 \equiv \varepsilon \) is the mean permittivity of the layer, and \( \delta \varepsilon_2 = \delta \varepsilon(x, y) \) is the inhomogeneity of the permittivity of the layer (which is assumed to depend solely on the horizontal coordinates). The lower layer is semi-infinite and has permittivity \( \varepsilon_3 \). The upper semispace, which has permittivity \( \varepsilon_1 = 1 \), contains a radiator and a receiver at point \( A \) at distance \( R_0 \) from the medium. Using the Kirchhoff approximation for calculation of the reflected radiation, the expressions for the complex voltage amplitudes at the receiver output can be written as (see [1])

\[
\hat{U}_1 = \frac{Q_1}{\lambda^2 R_0^2} \int G^2(\alpha, \beta) \exp\{-2ik_1\hat{R}_0\} \exp\{2ik_1h_1\} F \, dx dy,
\]

(1)

where \( Q_1 = A_a(2P_aR_a)^{1/2} \), the quantities \( G(\alpha, \beta), A_a, \) and \( R_a \) are the directivity pattern, the effective area, and the active resistance of the antenna, respectively, \( P_a \) is the power supplied to the antenna, \( \hat{R}_0 = (x^2 + y^2 + R_0^2)^{1/2} \) is the distance from the antenna to the integration point, \( k_1 = 2\pi/\lambda_1 \) is the wavenumber in the upper semispace, and \( F \) is the reflectivity from the two-layer medium, which is given by

\[
F \approx \frac{R + ke^{2i\varphi}}{1 + kRe^{2i\varphi}} = R + \sum_{m=1}^{\infty} (-1)^{m+1}(kR)^m(R^{-1} - R)e^{2im\varphi} = \sum_{m=0}^{\infty} f_m e^{2im\varphi}.
\]

(2)

Here, \( R \) and \( k \) are the Fresnel reflectivities from the upper and lower boundaries, respectively. The phase difference in the layer is given by

\[
\varphi \approx -k_1(\varepsilon)^{1/2}(H + h_1 - h_2 + H \delta \varepsilon/(2\varepsilon)).
\]

(3)
In Eq. (2), we neglect the variations of $R$ and $k$ due to the inhomogeneous permittivity of the layer. In Eq. (3), we neglect terms of higher order than the first term with respect to $\delta \varepsilon$ and $h_1, 2$. In so doing, we assume that the following conditions are satisfied ($V \equiv R, k$):

$$\left| (\partial V / \partial \varepsilon) \delta \varepsilon \right| \ll |V|, \quad |\delta \varepsilon / \varepsilon| \ll 1,$$

$$k_1 h_{1,2} \delta \varepsilon / (\varepsilon)^{1/2} \ll \pi,$$

$$k_1 H (\delta \varepsilon)^2 / (\varepsilon)^{3/2} \ll 4\pi.$$ 

Note that when conditions (5) and (6) are met, the terms in (3) that are proportional to $h_1$, $h_2$, and $\delta \varepsilon$ still can markedly exceed $\pi$. Let us calculate the two-frequency correlation function $\langle \hat{U}_1 \hat{U}_2 \rangle$ ($k_2 = 2\pi / \lambda_2$)

$$\langle \hat{U}_1 \hat{U}_2 \rangle \equiv \sum_{m,n=0}^{\infty} \langle \hat{U}_{1,m} \hat{U}_{2,n} \rangle = \frac{Q_1^2}{\lambda_1^2 \lambda_2^2 R_0^4} \int dz dy \int dz' dy' G^2(\alpha, \beta) G^2(\alpha', \beta') \times$$

$$\times \exp \{-2i(k_1 \bar{R}_0 + k_2 \bar{R}_0')\} \langle \exp \{2i(k_1 h_1 + k_2 h_1')\} F(k_1) F'(k_2) \rangle =$$

$$= \frac{Q_1^2}{\lambda_1^2 \lambda_2^2 R_0^4} \int dz dy \int dz' dy' G^2(\alpha, \beta) G^2(\alpha', \beta') \exp \{-2i(k_1 \bar{R}_0 + k_2 \bar{R}_0')\} \times$$

$$\times \sum_{m,n=0}^{\infty} f_m f_n \exp \{-2i\sqrt{\varepsilon} H (mk_1 + nk_2)\} \langle \exp \{2ik_1[(1-m\sqrt{\varepsilon})h_1 + m\sqrt{\varepsilon}h_2 -$$

$$- m\sqrt{\varepsilon} H \delta \varepsilon / (2\varepsilon)] + 2ik_2[(1-n\sqrt{\varepsilon})h_1' + n\sqrt{\varepsilon}h_2' - n\sqrt{\varepsilon} H \delta \varepsilon' / (2\varepsilon)]\rangle \rangle.$$

The angular brackets in (7) denote the statistical averaging over random realizations of $h_1$, $h_2$, $\delta \varepsilon / (2\varepsilon)$, $h_1'$, $h_2'$, and $\delta \varepsilon' / (2\varepsilon)$. Note that $\langle \hat{U}_1 \hat{U}_2 \rangle$ is derived from $\langle \hat{U}_1 \hat{U}_2 \rangle$ by substitution of $-k_2$ for $k_2$. In averaging Eq. (7), we assume that the random quantities are statistically homogeneous and obey the Gaussian distribution law, and we use the expression

$$\langle \exp \{2i \vec{\alpha} \cdot \vec{s} \rangle = \exp \{-2 \vec{\alpha} \cdot \vec{\bar{B}} \cdot \vec{\alpha} \rangle,$$

where the components of the vectors $\vec{s}$ and $\vec{\alpha}$ are given by $s_1 = h_1$, $s_2 = h_2$, $s_3 = \delta \varepsilon / (2\varepsilon)$, $s_4 = h_1'$, $s_5 = h_2'$, $s_6 = \delta \varepsilon' / (2\varepsilon)$, $\alpha_1 = k_1(1 - m \sqrt{\varepsilon})$, $\alpha_2 = k_1 m \sqrt{\varepsilon}$, $\alpha_3 = -k_1 m \sqrt{\varepsilon} H$, $\alpha_4 = k_2(1 - n \sqrt{\varepsilon})$, $\alpha_5 = k_2 n \sqrt{\varepsilon}$, and $\alpha_6 = -k_2 n \sqrt{\varepsilon} H$; $\vec{\bar{B}}$ is a covariant matrix with the elements $B_{ij} = \rho_{ij} \sigma_{ij}$ ($i, j = 1, \ldots, 6$); $\rho_{ij}(z-z', y-y') = \langle s_i s_j \rangle / \langle \sigma_i \sigma_j \rangle$ are the correlation factors of $s_i$ and $s_j$; $\sigma_1^2 = \sigma_2^2 = \sigma^2_{h_1}$ and $\sigma_3^2 = \sigma_4^2 = \sigma^2_{h_2}$ are the dispersions of the inhomogeneities of the upper and lower boundaries, respectively; $\sigma_5^2 = \sigma^2_{\varepsilon}$ is one quarter of the dispersion of the relative permittivity inhomogeneities in the layer.

Assuming that the antenna pattern $G(\alpha, \beta)$ has the form [2] (its axis is oriented vertically downward)

$$G(\alpha, \beta) = \exp \left\{ -1.38 \left( z^2 + y^2 \right) / (\theta_0^2 R_0^2) \right\},$$

from Eq. (7) we find the following expression for the two-frequency correlation function (putting $Q_1^2(k_1) = Q_1^2(k_2) = Q_1^2$):

$$\langle \hat{U}_1 \hat{U}_2 \rangle = \frac{Q_1^2}{\lambda_1^2 \lambda_2^2 R_0^4} \int dz dy \int dz' dy' \exp \left\{ -2.76 \left[ (z^2 + y^2) / (R_0^2 \theta_0^2 (k_1)) + $$

$$+ \left( (z')^2 + (y')^2 \right) / (R_0^2 \theta_0^2 (k_2)) \right\} - 2i(k_1 \bar{R}_0 + k_2 \bar{R}_0') \times$$

$$\times \sum_{m,n=0}^{\infty} f_m f_n \exp \{-2i\sqrt{\varepsilon} H (mk_1 + nk_2)\} \exp \{ -\gamma(m, n) - \Delta(m, n) \}.$$