OPTIMAL ESTIMATION OF THE PARAMETERS OF A NONSTATIONARY RANDOM PULSE TRAIN IN DISCRETE TIME

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The problem of optimal estimation of the parameters of a nonstationary sequence of random pulses observed additively with the noise is considered with the step change in average pulse repetition rate taken into account. A recurrent algorithm is derived for optimal estimation of the pulse repetition rate and time of its jump in real time. The optimal assessment of the Bernoulli trial parameters is shown as an example. The results of computer simulation experiments are presented.

1. INTRODUCTION

In many problems, pulse perturbation models [1–3] are used for description of the signal read-through situation. Sequences of pulse perturbations of a priori known shape but with random amplitude and time of occurrence are useful, for example, for tracing maneuvering objects, detecting abrupt changes in the signal and noise parameters in radio communication, following up signals in optical communication, checking measurement results in technical and medical diagnostic systems, etc. Using the theory of Markovian conventional processes [4, 5], the problem of optimal signal filtering in the joint action of pulse and noise perturbations was solved in [6, 7]. In the present paper, this theory is used for optimal estimation of the parameters of a nonstationary flow of pulse perturbations in discrete time, such as the mean pulse repetition rate and the random time of its variation.

2. STATEMENT OF THE PROBLEM

Suppose that the observed random process in discrete time is described by

$$\mathbf{z}_{k+1} = F_k \mathbf{z}_k + G_k \mathbf{\xi}_k + \sum_{i=1}^{\infty} \tilde{A}_i \delta_{kt_i}, \quad (k = 0, 1, 2, \ldots),$$

where $F_k$ and $G_k$ are the given matrices; $\{\mathbf{\xi}_k\}$ is a sequence of independent vectors with probability densities $p_{\mathbf{\xi}}(\mathbf{\xi}_k)$, which describes the additive noise perturbation; $\tilde{A}_i$ is the amplitude vector; $t_i$ is the time of occurrence of the $i$th pulse perturbation; $\delta_{kt_i}$ is Kronecker’s symbol.

Assume that the amplitudes of the pulse perturbations are statistically independent and have identical a priori probability densities $P_{\tilde{A}}(\tilde{A}_i)$ for different $i$ and the pulse occurrence statistics does not depend on the amplitude and is described by the Bernoulli trial parameters, i.e., the probability $\lambda(\tau, k)$ of occurrence of a current pulse at instant $k$ does not depend on the times of occurrence of the previous pulses and undergoes sudden change at instant $\tau$,

$$\lambda(\tau, k) = \begin{cases} \lambda_0, & \text{for } k < \tau, \\ \lambda_1, & \text{for } k \geq \tau, \end{cases}$$

where $\lambda_0$ and $\lambda_1$ have the meaning of average frequencies of occurrence of pulse perturbations before and
after the jump time \( \tau \), respectively. We assume that \( A_0 \) and \( A_1 \) are continuous random quantities with a priori known probability densities \( P(x_0|A_0) \) and \( P(x_1|A_1) \), defined for \( 0 \leq A_0 \leq 1 \) and \( 0 \leq A_1 \leq 1 \); \( r \) is a discrete random quantity with a priori known distribution function \( P_r(\tau) \) of potential times of jump occurrence, \( \tau \), for \( \tau = 0,1,2,\ldots \).

Note that in this model of pulse perturbations, the conventional a priori probabilities of ordered times of occurrence of current pulses for fixed \( \tau \) and times of occurrence of the previous pulses are given by

\[
P(t_1 | \tau) = \lambda(\tau, t_1) \prod_{k=0}^{t_1-1} \left[ 1 - \lambda(\tau, k) \right], \quad (t_1 = 0,1,\ldots),
\]

\[
P(t_n | t_{n-1}, t_{n-2}, \ldots, t_1, \tau) = P(t_n | t_{n-1}, \tau) = \lambda(\tau, t_n) \prod_{k=t_{n-1}+1}^{t_n-1} \left[ 1 - \lambda(\tau, k) \right],
\]

\((t_n = t_{n-1} + 1, t_{n-1} + 2,\ldots; \quad n = 2,3,\ldots)\). \quad (3)

Specifically, if the jump in the pulse perturbation statistics has not occurred before \( t_n \) \((t_n < \tau)\), then \( \lambda(\tau, k) = \lambda_0 \), and we find, from (3), the geometrical distributions for conventional probabilities of jump instants

\[
P(t_1 | \tau) = \lambda_0(1 - \lambda_0)^{t_1},
\]

\[
P(t_n | t_{n-1}, \tau) = \lambda_0(1 - \lambda_0)^{t_n-t_{n-1}-1}, \quad (t_1 \geq 0; \quad t_n > t_{n-1}; \quad n \geq 2).
\]

It is seen that expressions (3) and (3') represent the conventional probabilities of times \( t_i \) not only for a fixed value of \( \tau \) but also for fixed values of \( \lambda_0 \) and \( \lambda_1 \). Since the parameters \( \lambda_0, \lambda_1, \) and \( \tau \) are assumed to be random themselves, the point process describing pulse occurrences is a process with double randomness \([8, 9]\). Note that the observed process \( z_k \) given by Eq. (1) is Markovian only in combination with the random parameters \( \lambda_0, \lambda_1, \) and \( \tau \) in this formulation of the problem. However, if it is known a priori that, for example, \( k < \tau \) and \( \lambda(\tau, k) = \lambda_0 \) is a fixed constant, then the random process \( z_k \) will possess the Markovian property itself and its probability density in discrete time will be described by an equation similar to the Kolmogorov-Teller equation for the processes in continuous time \([7]\).

The objective of this paper is to find an algorithm for optimal estimation of the unknown parameters \( \lambda_0, \lambda_1, \) and \( \tau \) of a pulse perturbation sequence, using the results of observation of \( \{z_0, z_1, \ldots, z_T\} = z_0^T \) of the process \( z_k \) in the time interval \([0, T]\), and to investigate the accuracy of the optimal estimates of \( \hat{\lambda}_0, \hat{\lambda}_1, \) and \( \hat{\tau} \).

3. ALGORITHM FOR OPTIMAL ESTIMATION OF PULSE REPETITION RATE

The most complete information about \( \lambda_0 \) and \( \lambda_1 \) is contained in the posterior probability density \( P(\lambda_0, \lambda_1 | z_0^T) \equiv W(\lambda_0, \lambda_1; T) \) of the set of these parameters. In particular, the optimal root-mean-square estimates of \( \hat{\lambda}_0 \) and \( \hat{\lambda}_1 \) can be found as the mathematical expectations of the probability density

\[
\hat{\lambda}_j \equiv \lambda_j(T) = \int_0^1 d\lambda_0 \int_0^1 d\lambda_1 \lambda_j W(\lambda_0, \lambda_1; T), \quad (j = 0,1).
\]

The accuracy of estimation can be characterized by the dispersions

\[
D_j \equiv D_j(T) = \int_0^1 d\lambda_0 \int_0^1 d\lambda_1 (\lambda_j - \hat{\lambda}_j)^2 W(\lambda_0, \lambda_1; T), \quad (j = 0,1).
\]

Let us represent the posterior probability density \( W(\lambda_0, \lambda_1; k) \) for \( k = 0,1,\ldots, T \) as the sum

\[
W(\lambda_0, \lambda_1; k) = p_0(k) W_0(\lambda_0, \lambda_1; k) + p_1(k) W_1(\lambda_0, \lambda_1; k),
\]

(6)