2) B-derivatives \( v_k(t) \) with respect to \( \mu_0^k \), which are solutions of the system

\[
\frac{dv}{dt} = A(t)v + g_k(t), \quad t \neq \tau_k, \quad \Delta v|_{t=\tau_k} = P_i v + J_i
\]

with initial conditions \( v_k(t_0) = 0, k = 1, \ldots, m. \)

LITERATURE CITED


AVERAGING IN PARABOLIC SYSTEMS SUBJECT TO WEAKLY DEPENDENT RANDOM ACTIONS.

THE \( L_2 \)-APPROACH

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The first initial-boundary problem for a parabolic equation with a small parameter under external action described by some random process satisfying an arbitrary condition of weak dependence is considered. Averaging of the coefficients over a time variable is carried out. The existence of a generalized solution for the initial stochastic problem as well as for the problem with an "averaged" equation which turns out to be deterministic is assumed. Exponential bounds of the type of the well-known Bernstein inequalities for a sum of independent random variables are established for the probability of the deviation of the solution of the initial equation from the solution of the "averaged" problem.

Consider on \( \Theta_{T/\varepsilon} = [0, T/\varepsilon] \times G \) the first initial-boundary problem

\[
\frac{\partial u_\varepsilon}{\partial t} = \varepsilon [\mathcal{L}_\varepsilon u_\varepsilon + A(t, x, U_\varepsilon) + \sigma(t, x, U_\varepsilon, \eta(t))] + \beta(t, x),
\]

\[
U_\varepsilon(t, x)|_{t=\tau_k} = q_k(x), \quad U_\varepsilon(t, x)|_{t+\varepsilon} = \Phi(t, x).
\]


Here $G$ is a bounded open set in $\mathbb{R}^n$, $\partial G$ is its sufficiently smooth boundary, $\varphi \in L_2(G)$, $\varepsilon > 0$ is a small parameter, $\eta(t)$ is a centered random process.

Below we shall always assume that the coefficients of Eq. (1), the boundary and the values on the boundary are such that there exists a unique generalized $[1, 2]$ solution of problem (1).

In the present case

$$\mathcal{A}_x V = \sum_{i,j=1}^n \frac{\partial}{\partial x_i} \left[ a_{ij}(t, x) \frac{\partial V}{\partial x_j} \right] + \sum_{i=1}^n b_i(t, x) \frac{\partial V}{\partial x_i}.$$

Consider along with (1) the initial-boundary problem

$$\frac{\partial U_0}{\partial t} = \mathcal{A}_x U_0 + A(x, U_0),$$

$$U_0(t, x)|_{t=0} = \varphi(x), U_0(t, x)|_{x \in \partial G} = \Phi(t, x),$$

where

$$\mathcal{A}_x V = \sum_{i,j=1}^n \frac{\partial}{\partial x_i} \left[ \tilde{a}_{ij}(x) \frac{\partial V}{\partial x_j} \right] + \sum_{i=1}^n \tilde{b}_i(x) \frac{\partial V}{\partial x_i},$$

the coefficients $\tilde{a}_{ij}(x), i, j = 1, \ldots, n$, $\tilde{b}_i(x), i = 1, \ldots, n$, $\tilde{A}(x, z)$ are obtained respectively from $a_{ij}(t, x), i, j = 1, \ldots, n$, $b_i(t, x), i = 1, \ldots, n$, $A(t, x, z)$ by means of averaging over the time variable, i.e., the following is assumed:

I. There exist the limits

$$\lim_{T \to +\infty} \frac{1}{T} \int_0^T a_{ij}(t, x) \, dt = \tilde{a}_{ij}(x), \quad i, j = 1, \ldots, n,$$

$$\lim_{T \to +\infty} \frac{1}{T} \int_0^T b_i(t, x) \, dt = \tilde{b}_i(x), \quad i = 1, \ldots, n,$$

$$\lim_{T \to +\infty} \frac{1}{T} \int_0^T A(t, x, z) \, dt = \tilde{A}(x, z).$$

Concerning the random process $\eta(t)$ we assume that:

II. The random process $\eta(t)$ is such that $\mathbb{E} |\eta(t)| < +\infty$,

$$\mathbb{E} \left[ \sup_{0 \leq t \leq T} \left| \int_0^t \eta(s) \, ds \right| > R \right] \leq C_1 \exp \left( -C_2 R^\alpha \right) + r_1(\varepsilon),$$

$$\mathbb{E} \left[ \int_0^T \left( \left| \eta(s) \right| - M \left| \eta(s) \right| \right) \, ds \geq r \right] \leq C_3 \exp \left( -C_4 r^\beta \right) + r_2(\varepsilon),$$

where $C_i > 0$, $i = 1, \ldots, 4$, $\alpha > 0$, $\beta > 0$, $r_1(\varepsilon) \to 0$ as $\varepsilon \to 0$, $r_2(\varepsilon) \to 0$ as $\varepsilon \to 0$.

III. The nonrandom coefficients $a_{ij}(t, x), i, j = 1, \ldots, n$, $b_i(t, x), i = 1, \ldots, n$, are such that $\varepsilon \sum_{i=1}^n \tilde{a}_{ij}(x) \tilde{z}_i \tilde{z}_j$ for any point $z = (z_1, \ldots, z_n) \in \mathbb{R}^n$. We shall assume