\[
\frac{1}{4V\varepsilon} \int_0^t \sin(2s/\varepsilon + x)e^{-s}ds = \frac{V\varepsilon}{4} \left[ \cos x - \cos(2t/\varepsilon + x)e^{-t} - \int_0^t \cos(2s/\varepsilon + x)e^{-s}ds \right],
\]

it is easy to arrive at the inequalities

\[
\rho \leq \frac{9\pi}{V\varepsilon}, \quad \int_0^T \sigma_t \, dt \leq 4n, \quad \int_0^T \sigma_s \, dt \leq \pi, \quad \int_0^T \beta_t \, dt \leq 124\pi n.
\]

Thus, all the conditions of the theorem are fulfilled and the bound (3) is valid.

LITERATURE CITED

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ASYMPTOTIC PROPERTIES OF CORRELATION BOUNDS IN FUNCTIONAL SPACES. II

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This paper is an extension of [11]. Starting from the results of our first paper we prove by inclusion theorems that bounds for the correlation function of a stationary Gaussian process in the space of continuous functions with weight are strongly consistent and asymptotically normal. We construct the simplest functional confidence intervals in these spaces for the indeterminate correlation function.

This paper is an extension of the work [1] and the numbering of the formulas and references in this paper continues the numbering there.

A Study of the Properties of the Statistic \( \hat{B}_n \) in the Space \( C_0 \) by the Method of the Enclosed Hilbert Space. We recall that \( C_0(\mathbb{R}; [0, +\infty)) \) denotes the Banach space of continuous functions \( \varphi: [0, +\infty) \to \mathbb{R} \), continuous on \( [0, +\infty) \) and satisfying the condition \( \lim_{u \to +\infty} q(u)\varphi(u) = 0 \), and with norm \( \|q\|_{C_0(\varphi)} = \sup_{u \geq 0} q(u)\varphi(u) \). We will consider the asymptotic properties of the bound (1) in the spaces \( C_0(\varphi; [0, +\infty)) \). We put \( m = 1 \). Suppose \( X(t), t \geq 0 \) is a separable measurable modification of the stationary Gaussian process, \( \{X_k(t), t \geq 0\}, k = 1, 2, \ldots \), is a collection of independent copies of the random process \( X \). In formula (1) we put \( \delta_n = [0, T_n] \) where \( T_n \geq T_1 > 0, n = 1, 2, \ldots \), \( \lim_{n \to +\infty} T_n = +\infty \). Then the bound (1) takes the form

\[
\hat{B}_n(h) = \frac{1}{n} \sum_{k=1}^{n} \frac{1}{T_n} \int_0^{T_n} X_k(t) X_k(t + h) \, dt,
\]

and it is sufficient to consider it for \( h \geq 0 \).

We will say that the function \( q: [0, +\infty) \to [0, +\infty) \) belongs to the class \( Q_1[0, +\infty) \) if:
1) \( q(u) \) is almost everywhere continuously differentiable on \( (0, +\infty) \);
2) \( \operatorname{mes}\{u: q(u) = 0 \text{ or } q'(u) = 0\} = 0 \);
3) \( \int_{\mathbb{R}^m} q(u) \, du < +\infty, \int_{\mathbb{R}^m} |q'(u)| \, du < +\infty. \)

We bring in a scalar product for continuously differentiable functions \( \varphi_1, \varphi_2: [0, +\infty) \to \mathbb{R} \)

\[
\langle \varphi_1, \varphi_2 \rangle = \int_0^{+\infty} \varphi_1'(u) \varphi_2'(u) \, du + \int_0^{+\infty} \varphi_1'(u) \varphi_2(u) \, du + \varphi_2(0) \varphi_1(0)
\]

and the corresponding norm

\[
\| \varphi \|_H = \langle \varphi, \varphi \rangle^{1/2}_H.
\]

We will denote by \( H^{(1)}(q) \) the closure of the set \( \{ \varphi \in C_0([0, +\infty)): \varphi' \in C_0([0, +\infty)), \| \varphi \|_H < \infty \} \)
in the norm \( \| \cdot \|_H \). If the weight function \( q \) lies in \( Q_1[0, +\infty) \), then the space \( H^{(1)}(q) \) with scalar product (19) is a separable Hilbert space imbedded in \( C_0([0, +\infty)) \). Further, for an arbitrary function \( \varphi \in H^{(1)}(q) \) we have

\[
\| \varphi \|_{C_0(q)} \leq K_1 \| \varphi \|_H,
\]

where

\[
K_1 = \left[ 3 \max \left\{ \int_0^{+\infty} q(u) \, du, \int_0^{+\infty} |q'(u)| \, du, q^2(0) \right\} \right]^{1/2}.
\]

**Theorem 5.** Suppose \( X(t), t \geq 0 \) is a stationary Gaussian random process with finite second spectral moment

\[
\omega_2 = \int_{-\infty}^{+\infty} |\lambda|^2 \phi(\lambda) \, d\lambda < +\infty.
\]

Then for an arbitrary function \( q \in Q_1[0, +\infty) \) the bounds \( \hat{B}_n, n = 1, 2, \ldots \), are random elements of the space \( H^{(1)}(q) \) and \( P\{ \lim_{n \to +\infty} \| \hat{B}_n - B \|_H = 0 \} = 1 \), i.e., \( \hat{B}_n \) is strongly consistent in \( H^{(1)}(q) \).

**Proof.** We will find a bound for the expression

\[
E \| \hat{B}_n - B \|_H^2 = \int_0^{+\infty} \int_0^{+\infty} \int_0^{+\infty} |q'(u)| (\hat{B}_n(u) - B(u))^2 \, du + \int_0^{+\infty} q(u)(\hat{B}_n(u) - B(u))^2 \, du
= \int_0^{+\infty} (\hat{B}_n(u) - B(u))^2 \, du + q^2(0)(\hat{B}_n(0) - B(0))^2.
\]

We will denote by \( A_{ij}, i, j = 1, \ldots, 3, i \leq j \) the terms obtained after expanding out the parentheses in the right hand side of (24) by multiplying the \( i \)-th term by the \( j \)-th. We will write out the bounds for these terms. We have

\[
A_{11} = \int_0^{+\infty} |q'(u)| (\hat{B}_n(u) - B(u))^2 \, du.
\]

The bound of this term follows from inequality (13) with \( |q'(u)| \) as weight function. Then

\[
A_{11} \leq 12 \omega^4 \left( \frac{1}{n^2} + \frac{5}{n^2} \right) \left( \int_0^{+\infty} |q'(u)| \, du \right)^2.
\]