ON N. N. BOGOLYUBOV'S WORKS IN CLASSICAL
AND QUANTUM STATISTICAL MECHANICS

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A review of N. N. Bogolyubov's works in classical and quantum statistical mechanics is presented.

Introduction. Nicolai N. Bogolyubov is well known as one of the most prominent scientists of the twentieth century. His heritage affects the whole of advance in modern mathematics and theoretical physics. His works founded new directions in nonlinear mechanics, statistical physics, and quantum field theory.

The remarkable breadth of Bogolyubov's creative interests, the tremendous range and variety of those branches of mathematics, mechanics, and theoretical physics in which he worked, and the principal importance of Bogolyubov's fundamental results distinguish him among all other scientists. One can freely say that Bogolyubov is one of the most talented scientists of the twentieth century. In all fields of mathematics, mechanics, and theoretical physics in which he worked, Bogolyubov obtained fundamental results by overcoming serious and sometimes tremendous difficulties. His works may be regarded as classic, and their significance only increases as time goes by.

Bogolyubov is the author of hundreds of articles and dozens of monographs, his results are included in many monographs and textbooks. However, there is a lack of surveys in which not only Bogolyubov's results are listed and their significance is pointed out, but also the ideas of proofs and the history of a problem are presented in detail over all directions of his creative activities.

The authors' aim is to fill in this gap and to give such a nontypical survey of Bogolyubov's works in statistical mechanics.

The survey consists of an introduction and nine sections. Since Bogolyubov's works in statistical mechanics were the natural sequel of his research in nonlinear mechanics, Section 1 contains a brief review of the latter. The next eight sections include the most important of Bogolyubov's results in statistical mechanics. Below, we briefly present the contents of these sections.

Section 2 is devoted to the theorem on the existence of invariant measure for compact dynamical systems. Let us say a few words about the history of the question and the significance of this theorem for statistical mechanics.

In the late thirties, Bogolyubov's interests were changing gradually from nonlinear mechanics to the theory of dynamical systems and statistical mechanics. By that time, von Neumann and Birkhoff had proved the ergodic theorems connected with Boltzmann's ergodic hypothesis on the coincidence of averages with respect to time and with respect to the phase space. In proving these theorems, the assumption of the existence of invariant measure for dynamical systems was essentially employed. For Hamiltonian systems, the required measure is the volume of the phase space. However, it was unknown whether the invariant measure exists for any dynamical system. A positive answer to this question was given by Bogolyubov and Krylov, who proved the fundamental theorem on the existence of invariant measure for a compact dynamical system. They introduced the concept of ergodic sets and showed that the invariant measure can be decomposed into invariant measures that are concentrated on ergodic sets and cannot be decomposed. These results were extended to stochastic systems for which the ergodic theorems were also proved. For this purpose, fine properties of completely continuous positive operators were used.

In Section 3, we consider a model which demonstrates the transition to statistical equilibrium in a system connected with a heat bath. This model is an oscillator interacting with a large system which is situated in a statistical equilibrium state (this is the heat bath). It is shown that if the number of particles in the heat bath infinitely increases, and time tends to infinity, then the state of the system approaches equilibrium. This was the first example of mathematically rigorous justification of the conventional hypothesis that a system connected with a heat bath approaches equilibrium.

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In Section 4, we present a review of works in classical statistical mechanics. We discuss, mainly, the papers dealing with the study of equations for a sequence of distribution functions and with justification of the thermodynamic limit procedure. In the monograph "Problems of Dynamical Theory in Statistical Physics," the classical statistical mechanics is formulated in terms of sequences of distribution functions and equations for these, which are now called Bogolyubov equations (or BBGKY hierarchy). As a matter of fact, a new concept of the state of systems of statistical mechanics was introduced, and the evolution of state was described by the Bogolyubov equations. The states of infinite systems were determined in terms of the states of finite systems by the thermodynamic limit procedure, and the justification of the thermodynamic limit transition was outlined for the first time in the world scientific literature.

On the basis of the equations for distribution functions and the fundamental concept of cluster property, Bogolyubov derived the Boltzmann equations without employing the hypothesis of "molecular chaos" and the Vlasov and Landau equations. The notion of time hierarchy and different evolution stages (chaotic, kinetic, and hydrodynamic) connected with this were exactly formulated. Thus, for the first time, a consistent method was introduced for rigorous construction of equations of physical kinetics for a broad class of neutral and charged systems.

These Bogolyubov papers gave modem form to classical statistical mechanics and, in fact, opened a new (next to Maxwell, Boltzmann, and Gibbs), Bogolyubov stage in the progress of classical statistical mechanics.

Bogolyubov was the first one who stated the problem of justification of the thermodynamic limit procedure for equilibrium states and reduced it to the functional-analysis problem concerning the existence of a solution to an operator equation for distribution functions and its limit properties. First, systems of repelling particles were considered. Then, this problem was completely solved for general systems with stable short-range interaction in canonical ensemble. The method of justification of the thermodynamic limit procedure suggested by Bogolyubov in 1949 was reanimated in the sixties and applied to the case of grand canonical ensemble.

In Section 5, we discuss the papers dealing with the investigation of the states of quantum systems. In the monograph "Lectures in Quantum Statistics," Bogolyubov introduced a new concept of states of infinite systems in quantum statistical mechanics in terms of an infinite sequence of statistical operators; the evolution of state is described by equations for statistical operators (Bogolyubov equations). In this monograph, a new approach to the approximate second quantization was elaborated and applied to the polar theory of metals and the theory of magnetism.

In this section, we also consider the problem of justification of the thermodynamic limit procedure for equilibrium states of quantum systems and show that the methods developed by Bogolyubov for classical equilibrium systems can be applied to quantum systems, too.

In Section 6, we present Bogolyubov's microscopic theory of superfluidity. Before Bogolyubov's works were published, Landau's phenomenological theory based on the assumption concerning the shape of the spectrum of elementary excitations had existed. On the basis of the general Hamiltonian for Bose-systems, under the assumption that a macroscopic number of particles is situated in the ground state with momentum zero (and hence, the operators of creation and annihilation of particles with momentum zero are c-numbers), Bogolyubov obtained an approximating Hamiltonian — a quadratic form with respect to operators of creation and annihilation. The standard perturbation theory was inapplicable in this case because of the strong interaction of particles with opposite momenta. Consequently, the Hamiltonian was diagonalized by the use of canonical transformation (Bogolyubov \( u-v \)-transformation) and, definitely out of the scope of perturbation theory, the spectrum of elementary excitations was found.

As a matter of fact, by decomposing field operators into \( c \)-number and operator parts, Bogolyubov introduced in the quantum theory the method of spontaneous break of symmetry for the systems with degenerate ground state. Decades later, this method was reanimated in the quantum field theory.

In Section 7, the theory of superconductivity is presented. In 1957, Bardeen, Cooper, and Schrieffer suggested a model for explaining superconductivity. For this model, according to which only electrons with opposite momenta and spins interact with each other, they determined the ground state, its energy, and the spectrum of elementary excitations. At the same time, Bogolyubov independently worked out a complete theory of superconductivity based on an investigation of systems of interacting electrons and phonons. By generalizing his method of canonical transformations for Fermi-systems and introducing the principle of compensation of dangerous diagrams, Bogolyubov determined the ground state consisting of paired electrons with opposite momenta and spins, its energy, and the energy of elementary excitations. It was shown that the superconductivity phenomenon consists in pairing of electrons with