In analogous manner one can show that the ansatzen 2), 3) in (12), too, can be obtained by using the conditional symmetry of Eq. (1).

In conclusion we remark that the ansatzen (12) reduces to a system of ODEs the general spinor equations

\[ \{i\gamma_{\alpha} \partial_{\mu} - (\bar{\psi}\gamma)_{1/2}\bar{f}_1 (\bar{\psi}\gamma\gamma_{1/2}) + \bar{f}_2 (\bar{\psi}\gamma(\bar{\psi}\gamma\gamma_{1/2})_{1/2}) \} \bar{\psi} = 0, \]

where \( \bar{f}_2 \in C^1(\mathbb{R}, \mathbb{C}) \).

LITERATURE CITED


THEOREM ON GROUPS OF FINITE SPECIAL RANK

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It is proved that groups of finite special rank in the minimal class of groups containing the class of periodic, locally graduated groups and closed under the formation of local systems, sub-Cartesian products, and normal series are almost hyper-Abelian. In particular, a group of matrices over an arbitrary commutative associative ring with unity having finite special rank is almost hyper-Abelian. This result extends a well-known theorem of V. P. Platonov on the almost solvability of a linear group of finite special rank.

One of the fundamental concepts in group theory is that of the special rank of a group, which was introduced by Mal'tsev [1]. There are many important results connected with groups of finite special rank, reflected in a number of well-known monographs on group theory (see [2-6]). In particular, it is known that a locally solvable group of finite special rank is an extension of a hypercentral (or ZA-) group by an almost Abelian group and that the class of such groups coincides with the class of RA*-groups of finite special rank (M. I. Kargapolov; see, e.g., [4, p. 178]). A hypercentral group of finite special rank is an extension of a direct product of Chernikov p-groups with various primes p by a nilpotent group (A. I. Mal'tsev and N. N. Myagkova; see, e.g., [4, p. 38]). A periodic group of finite special rank need not be locally finite [7], but if it is locally finite, then it is almost locally solvable [8]. It is also known that a linear group of finite special rank is almost solvable (V. P. Platonov; see, e.g., [9], Theorem 10.9). (Recall that a group almost possesses some property if it has an invariant subgroup of finite index with that property.)

The main result of the present paper is the following theorem, which establishes that groups of finite special rank in a very extensive class of groups are almost hyper-Abelian.

Recall that a locally graduated group is a group in which each nonidentity, finitely generated subgroup has a subgroup of finite index different from 1. Note that the class of periodic, locally graduated groups is already very extensive.

**Theorem.** Suppose $\mathcal{X}$ is the minimal class of groups containing the class of all periodic, locally graduated groups and closed under the formation of local systems, sub-Cartesian products, and ascending and descending normal series. Then any group in $\mathcal{X}$ of finite special rank is almost hyper-Abelian, almost locally solvable, and, if periodic, is locally finite.

(Ascending and descending normal series of a group have the same meaning as in [2, 10].)

We mention some classes of groups contained in $\mathcal{X}$: the classes of locally finite, locally solvable, locally almost solvable, radical (in the sense of B. I. Plotkin), or residually finite groups; the classes of $\text{RN}^\infty$, $\text{RI}^\infty$, $\text{RA}$-, $\text{ZD}$-, or $\text{ZD}$-groups; the class of all linear groups; the class of all groups faithfully representable by automorphisms of finitely generated unital modules over commutative associative rings with unity (in particular, the class of all groups of matrices over such rings).

The above theorem includes the following result of the author (see [11], Theorem 1): A periodic, locally graduated group of finite special rank is locally finite.

Notation: $\mathcal{P}$ and $\mathcal{S}$ are the classes of periodic and solvable groups; $\mathcal{P}\mathcal{S}$ is the class of extensions of $\mathcal{P}$-groups by $\mathcal{S}$-groups; $<X^k>$, $k \in \mathbb{Z}$, is the subgroup generated by the $k$-th powers of the elements of the group $X$; $\mathcal{P}(X)$ is the subgroup generated by all invariant solvable subgroups of $X$; $F(X)$ is the Fitting subgroup of $X$, i.e., the subgroup generated by all invariant nilpotent subgroups of $X$ (if $X$ is finite, then $F(X)$ is nilpotent by Fitting's theorem (see, e.g., [6])); $O(x)$ is the largest invariant subgroup of odd order in the finite group $X$; $F^*(X)$ is the generalized Fitting subgroup of the finite group $X$; $a(X)$ is the exponent of $X$; $c(X)$ is the nilpotency length of the nilpotent group $X$; $r(X)$ is the special rank of $X$.

The proof of the theorem is preceded by several auxiliary results, which are also of independent interest.

**Lemma 1.** A locally graduated group of finite special rank and finite exponent has finite order not exceeding a constant depending only on the special rank and exponent of the group.

It is easy to prove the lemma by using the second part of its conclusion (see, e.g., [4], the proof of Theorem 8.16) and the obvious fact that a locally graduated group is finite if and only if the orders of all of its finite sections are bounded in the aggregate.

**Lemma 2.** In a group having finite special rank $r$, the intersection of all subgroups of index not exceeding some fixed natural number $m$ has finite index not exceeding a constant depending only on $r$ and $m$.

**Proof.** By Poincaré's theorem (see, e.g., [6]), any subgroup of finite index at most $m$ contains a subgroup, invariant in the whole group, of finite index dividing $m!$. By Remak's theorem (see, e.g., [6]), the factor group with respect to the intersection of all these invariant subgroups is residually finite. It remains to apply Lemma 1 to this factor group.

**Proposition 1.** Suppose a group $G$ has a local system $\Sigma$ of subgroups, each of which is an extension of a periodic group by an almost locally solvable group of finite special rank not exceeding some fixed $r$. Then $G/\mathcal{P}(G)$ is an almost solvable group of finite special rank at most $r$, the index of its solvable radical does not exceed some constant depending only on $r$, and this radical has a characteristic subgroup of finite index having a finite chain of characteristic subgroups with torsion-free Abelian factors.

**Proof.** We may assume with no loss of generality that $\mathcal{P}(G) = 1$ and $G \neq 1$. Any finitely generated subgroup of $G$ is contained in some subgroup belonging to $\Sigma$, hence possesses the same properties. We may therefore assume that $\Sigma$ is a system of certain finitely generated subgroups of $G$. By Lemma 10.39 of [4], there exists a natural number $t = t(r)$, depending only on $r$, such that the $t$-th commutant of any locally solvable group of finite special rank at most $r$ is a periodic hypercentral group that is decomposable into a product of Chernikov $p$-subgroups with various primes $p$. In particular, each subgroup in $\Sigma$ is an extension of a periodic group by an almost solvable group, the solvability length of whose solvable radical is at most $t$. Let us fix the number $t$. 

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