III. LOGIC, LANGUAGE, AND PHILOSOPHY

BOGUSŁAW WOLNIEWICZ

Logical Space and Metaphysical Systems

"Facts in logical space are the world."
Tractatus 1.13

Abstract. The paper applies the theory presented in "A Formal Ontology of Situations" (this journal, vol. 41 (1982), no. 4) to obtain a typology of metaphysical systems by interpreting them as different ontologies of situations. Four are treated in some detail: Hume's diachronic atomism, Laplacean determinism, Hume's synchronic atomism, and Wittgenstein's logical atomism. Moreover, the relation of that theory to the "situation semantics" of Perry and Barwise is discussed.

1. Situations

1.1. Logical Space. Consider a classic propositional language $L$. The logical space of $L$ is a metaphysical construction $SP$ comprising all possibilities expressible in $L$. These are situations (cf. [14]–[24]), and $S'$ is to be their totality. Thus, in a sense that will be defined, every possible situation $S$ is comprised in logical space:

$\forall S \in S' . \bigwedge_S S \in SP$. 
The situation presented by a proposition $\alpha$ is $S(\alpha)$. With Meinong (cf. [9]) we call it the objective of $\alpha$. Objectives are equal iff the corresponding propositions are strictly equivalent:

$$S(\alpha) = S(\beta) \text{ iff } \alpha \equiv \beta.$$ 

If real, situations are facts.

A contradiction has no objective in $S'$. To provide for it we augment $S'$ with the impossible situation $A$; i.e., $S = S' \cup \{A\}$. Thus $S : L \to S$ is a function mapping propositions into situations. But $A$ is beyond logical space: $\sim (A \in S')$; or to put it with Wittgenstein: "Contradiction is the outer boundary of propositions" (cf. [13], 5.143).

1.2. Elementary Situations. We start with a universe $SE''$ of elementary situations (cf. [24], of which this part is a summary). These correspond to conjunctions of atomic propositions: if $a$ is such a conjunction, then $S(a) = \{x\}$, for some $x \in SE''$. The universe consists of two parts: of the set $SE$ of proper (i.e. contingent) elementary situations, and of the two improper ones: the empty one $\emptyset$ and the impossible one $\lambda$. I.e., $SE'' = SE \cup \{\emptyset, \lambda\}$.

An elementary situation may obtain in another: $x \leq y$. This is a partial ordering such that $\emptyset \leq x \leq \lambda$, for any $x \in SE''$. Under it, $SE''$ is assumed to be a lattice of finite length. The join $x; y = \sup\{x, y\}$ corresponds to conjunction. The meet $x!y = \inf\{x, y\}$ has no obvious counterpart in $L$.

An elementary situation either verifies a given proposition of $L$, or falsifies it, or is neuter to it. Elementary situations verifying $\alpha$ are the verifiers of $\alpha$.

The minimal elements of $SE$, if any, are logical atoms (or states of affairs). The maximal ones are logical points (or possible worlds). Logical space is the totality of logical points (cf. [10] and [15]). Call Min $A = \{x \in A : \sim y < x, \text{ for all } y \in A\}$ the minimum of $A$, and likewise Max $A$ — its maximum. Then $SA = \text{Min } SE$, and $SP = \text{Max } SE$, provided $SE$ is not empty. Otherwise $SP = \{\emptyset\} = Q_0$, and $SA = \{\lambda\} = A$.

For any $w \in SP$, the set $R_w = \{x \in SE'' : x \leq w\}$ is a maximal ideal of $SE''$. With Loš [7] we call such sets realizations, and $R$ is to be their totality.

1.3. Verifiers, Objectives, and Loci of Propositions. Call sets of elementary situations $SE''$-sets for short. Situations are some of them: $S \subset P(SE'')$. There are several ways to determine $S$ precisely, but the following is the simplest one formally. Call two $SE''$-sets $V$-equivalent iff they intersect the same realizations. I.e.,

$$A \sim B \text{ iff } \bigwedge_{R \in R} (A \cap R = \emptyset \text{ iff } B \cap R = \emptyset).$$