ABSTRACT. Peter Geach proposed a substitutional construal of quantification over thirty years ago. It is not standardly substitutional since it is not tied to those substitution instances currently available to us; rather, it is pegged to possible substitution instances. We argue that (i) quantification over the real numbers can be construed substitutionally following Geach’s idea; (ii) a price to be paid, if it is that, is intuitionism; (iii) quantification, thus conceived, does not in itself relieve us of ontological commitment to real numbers.

1. INTRODUCTION

Can quantification over the real numbers be construed substitutionally? W. V. O. Quine has argued that it cannot be so construed. We argue, against Quine, that it can be so construed. We appeal to a conception of substitutional quantification put forth by Peter Geach. It might be thought that if quantification over the reals can be construed substitutionally, then real number theory need not be committed to the existence of real numbers. We reject this inference. Geach’s view itself is neutral with respect to any particular view regarding the ontology of real number quantification. On Geach’s construal of quantification, whether a type of quantification has ontological content depends on the uses of the terms for which the variables of that type of quantification go proxy. In connection with this, we make the point that one cannot read off the use of numerical terms from the fact that they sort with, e.g., proper names in respect to certain ranges of inferences. That is a point of commonality; but there is no established reason for supposing that, since ‘3’ and ‘Theaetetus’ are alike in respect to these inferences, their uses are sufficiently similar to mark them equally referential as well.

2. QUINE AGAINST MARCUS

Quine says that a true real number quantification \( \exists x \phi x \) can turn out false under a substitutional account, namely in case all the verifying real numbers are nameless ones. That there will be nameless ones is held to follow from the indenumerability of the real numbers and
the countability of the language. Thus Quine in considering Marcus’s substitutional construal of quantification writes:

... take the real numbers. On the classical theory, at any rate, they are indenumerable whereas the expressions, simple and complex, available to us in any language are denu-
merable. There are, therefore, among the real numbers, infinitely many none of which can be separately specified by any expression, simple or complex. Consequently, an existential quantification can come out true when construed in the ordinary sense, thanks to the existence of appropriate real numbers, and yet false when construed in Professor Marcus’s sense, if by chance those appropriate real numbers all happen to be severally unspecifiable.\(^1\)

3. GEACH’S IDEA

Geach writes:

I have said that a proposition beginning with a quantifier “For some ___” is true iff the proposition minus this quantifier could be read as a true proposition by taking the occurrence(s) of the letter ‘bound to’ the quantifier as if there were occurrence(s) of an actual expression belonging to the appropriate category. I do not mean here that the language we are using must already contain an actual expression, of the appropriate category, which, if substituted for the bound variable in the proposition minus the quantifier, would give us a true proposition: it is sufficient that we could coherently add such an expression to our language.\(^2\)

As an example Geach cites

For some \(x\), \(x\) is a pebble on the beach of Brighton

and says that the truth-value of this sentence

does not depend on anybody’s having given a proper name to such a pebble; it is enough that we could coherently add to our language a proper name of such a pebble.\(^3\)

In short, ‘\(x\) is a pebble on the beach of Brighton’ is a prototype standing for a certain possibility of sentence construction. The truth of the quantification of the prototype requires that it be possible to construct a sentence from that prototype which would turn out true.

The expressions of a language fall into different categories: proper names, predicates, connectives, etc. For each such category \(C\) it is possible to introduce a quantifier using variables pegged to \(C\). The formulas deploying such variables are prototypes standing for a certain possibility of sentence construction, and the truth of the quantification of such a prototype requires that it be possible to construct a sentence from that prototype which would turn out true.