AVERAGING OF APERIODIC PROBLEMS OF CONTROL OVER FRACTURE SURFACES

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An averaging method is justified for standard aperiodic systems with parameters and discontinuous right-hand sides under the condition of aperiodicity and applied to the problems of control over fracture surfaces.

Consider a standard system with discontinuous right-hand side

\[ \dot{x} = \varepsilon X(t, x, w) = \varepsilon \begin{cases} X_1(t, x) & \text{for } \Phi(t, x, w) \leq 0; \\ X_2(t, x) & \text{for } \Phi(t, x, w) > 0, \end{cases} \]

where \( \varepsilon > 0 \) is a small parameter, \( x \) and \( X \) are \( n \)-dimensional vector functions, \( w \in W \) is an \( m \)-dimensional vector of control parameters, and \( (t, x, w) \in Q = \{ t \geq 0, x \in D \subseteq \mathbb{R}^n, w \in W \subseteq \text{comp } \mathbb{R}^m \} \).

The problem is to find the values of the parameters \( w \in W \) for which the minimum value of a certain functional is attained, i.e.,

\[ J(w) = F(x(T)) \to \min_{w \in W}, \]

where \( T = L\varepsilon^{-1}, L > 0, \) and \( L = \text{const} \).

Problem (1), (2) is associated with the following averaged problem:

\[ \frac{d\xi}{d\tau} = \overline{X}(\xi, w), \quad \xi(0) = x_0, \]

\[ J_0(w_0) = F(\xi(L)) \to \min_{w \in W}, \]

where

\[ \overline{X}(x, w) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} X(t, x, w) dt, \quad \tau = \varepsilon t. \]

We introduce the sets

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\[ I_1(x, w) = \{ t \in [0, T] \mid \Phi(t, x, w) \leq 0 \}, \]
\[ I_2(x, w) = [0, T] \setminus I_1(x, w), \]
\[ \psi_1^*(x, w) = \lim_{t \to T} \frac{1}{t} \left( \text{mes} \{0 \leq \tau \leq t \mid \Phi(t, x, w) \leq 0\} \right), \]
\[ \psi_2^*(x, w) = \lim_{t \to T} \frac{1}{t} \left( \text{mes} \{0 \leq \tau \leq t \mid \Phi(t, x, w) > 0\} \right) \]

and define surfaces \( \varphi_i(x, w) = 0, \ i = 1, 2, \) such that \( \psi_i^* < 1, \ i = 1, 2, \) for \( \varphi_i(x, w) > 0 \) and \( \psi_i^* = 1, \ i = 1, 2, \) for \( \varphi_i(x, w) \leq 0, \ i = 1, 2. \)

The surfaces \( \varphi_i(x, w) = 0, \ i = 1, 2, \) define the domains

\[ D_1(w) = \{ x \in Q \mid \varphi_1(x, w) \leq 0 \}, \ w \in W, \]
\[ D_2(w) = \{ x \in Q \mid \varphi_2(x, w) \leq 0 \}, \ w \in W, \]
\[ D_3(w) = Q \setminus (D_1(w) \cup D_2(w)), \ w \in W. \]

**Theorem 1.** Suppose that the following conditions are satisfied in the region \( Q: \)

(i) \( X_i(t, x), \ i = 1, 2, \) are continuous in \( t, \) uniformly bounded by a constant \( M > 0, \) and satisfy the Lipschitz condition in \( x \) with a constant \( \lambda; \)

(ii) limit (5) exists uniformly in \( x; \)

(iii) the solution \( \xi_\tau(\tau) \) of system (3) is defined for all \( w \in W \) and \( \tau \geq 0 \) and lies, together with its \( p \)-neighborhood, in the region \( D; \)

(iv) the function \( \Phi(t, x, w) \) is continuous and piecewise smooth in \( t; \) the function \( \frac{\partial \Phi(t,x,w)}{\partial x} \) is continuous for \( w \in W; \)

(v) for all \( w \in W \) and \( x \in D, \) the equation \( \Phi(t, x, w) = 0 \) has simple roots \( t_j(x, w) \) and

\[ \frac{1}{T - t} K(x, w, t, T) \leq k, \]

where \( K(x, w, t, T) \) is the number of roots \( t_j(x, w) \) on the segment \( [t, T]; \) furthermore, the condition of "strict intersection" holds at the roots \( t_j(x, w). \)