In this paper we present a simpler proof of the following result obtained by Kratko: algorithmic unsolvability of the problem of recognition of the completeness of automata bases [2] and the existence of nonrecursive bases [3].

A basis $\mathcal{B}$ in the alphabet $A$ is a finite system of (finite) automata in the alphabet $A$. A circuit in the basis $\mathcal{B}$ is a logic network [1] designed from elements of the basis $\mathcal{B}$ with the aid of superposition and feedback (if it is applicable [1, 4]). It is well known that any circuit realizes a limited-deterministic (LD) operator. A basis $\mathcal{B}$ in the alphabet $A$ is said to be complete if for any LD operator in the alphabet $A$ it is possible to realize a circuit in this basis.

We shall assume the existence of a system of Post productions with an algorithmically unsolvable deducibility problem (see, for example, [5]).

A system of Post productions $\mathcal{P}$ is defined by its alphabet $A_1 = \{a_1, \ldots, a_h\}$ and a system of basic productions $(P_i)$:

$$P_iW \rightarrow WR_i \quad (1 \leq i \leq h),$$

where $P_i$ and $R_i$ are words in the alphabet $A_1$. Suppose that a word $Q$ begins with the word $P_i$. To realize over the word $Q$ a production $(P_i)$ signifies that we drop the initial subword $P_i$ of the word $Q$ and then adjoin the word $R_i$ to the right of the remaining word. It is assumed that the production $(P_i)$ is inapplicable to a word whose beginning is different from the word $P_i$.

We shall say that a word $U$ is deducible from a word $V$ in a system of productions $\mathcal{P}$ if there exists a finite chain $V_1, V_2, \ldots, V_r$ of words such that $V_1 = V$, $V_r = U$, and any word $V_{j+1}$ can be obtained from the previous word $V_j$ by a production $(P_i)$.

The following assertion is true ([5], p. 310).

**Assertion.** There exists a system of Post productions $\mathcal{P}$ in which the totality of words deducible from an appropriate fixed word $R_0$ is nonrecursive.

Let us note that there exist systems of Post productions with an algorithmically unsolvable deducibility problem in which all the words $P_i$ and $R_i$ are nonempty. In the following we shall have in mind precisely such a system of productions $\mathcal{P}$. Henceforth a word deducible from the word $R_0$ in a system of productions $\mathcal{P}$ will be simply called a deducible word.

In this paper we shall define automata with the aid of a system of instructions

$$q_i\bar{x}_r \rightarrow q_j\bar{y}_s,$$

where $q_i$ and $q_j$ are the automaton states, $\bar{x}_r$ is an input array, and $\bar{y}_s$ an output array. The automaton instructions have the following meaning. Suppose that at some instant the automaton is in a state $q_i$ and it receives an input array $\bar{x}_r$. Then in accordance with the instruction $q_i\bar{x}_r \rightarrow q_j\bar{y}_s$ (belonging to the system of instructions of the automaton) the output array of the automaton at this instant will be $\bar{y}_s$, whereas the state of the automaton at the next instant is $q_j$.

For shortening the notation of the system of instructions of automata we shall stipulate that not explicitly written instructions have the same right-hand side (placed after the symbol \( \Rightarrow \)).

Everywhere below we shall consider automata with one output. Let \( K \) and \( L \) be automata. An \( sK \) chain \((s \geq 0)\) is defined as a circuit consisting of \( s \) automata \( K \) each of whose outputs (apart from the last) is connected with the first input of the next one. The principal input of an \( sK \) chain (and of subsequently introduced chains) is defined as the first input of its first element. The output of the \( sK \) chain will be connected with the principal input of the \( tL \) chain. The thus-obtained circuit will be called an \((sK, tL)\) chain.

Let \( Q \) be a word in an alphabet. By the symbol \( \sigma_{iQ} \) we shall denote the \( i \)-th letter of the word \( Q \), and by the symbol \( |Q| \) its length (number of letters). An infinite sequence of letters in an alphabet will be called a superword. The periodic superword \( PQQ \ldots Q \ldots \) will be denoted by \( PQ^\infty \).

Let \( A_e = A_1 \cup \{\alpha, \gamma, \delta\} \). Everywhere below we shall consider automata in the alphabet \( A_0 \).

The code of a word \( R \) in an alphabet \( A_1 \) is defined as a superword of the form \( \alpha^P \) \( Ro^\infty \) for \( \nu \geq 1 \). Any superword of the form \( P\delta^\infty \), where \( P \) is a word in the alphabet \( A_0 \), will be called a \( \delta \) superword.

Let \( B_0 \) be an autonomous automaton without inputs that has \( |R_0| + 2 \) states defined by the following system of instructions:

\[
q_0 \rightarrow q_0; \quad q_i \rightarrow q_{i+1}; \quad \alpha \rightarrow q_0; \quad (1 \leq i \leq |R_0|);
\]

\[
q_R \rightarrow q_0; \quad \gamma \rightarrow q_0; \quad \delta \rightarrow q_0; \quad (1 \leq |R| - 2);
\]

\[
q_{|R|+\gamma} \rightarrow q_{|R|+\delta} \quad \Rightarrow q^\delta.
\]

Let us note that in a state other than \( q^* \) the automaton \( B_1 \) realizes a function that is strongly dependent on the input variable. In any of its states the automaton \( B_1 \) will transform the letter \( \delta \) into the letter \( \delta \) and go over (or remain) in the "bad" state \( q^* \); in the state \( q^* \) the automaton \( B_1 \) realizes the constant \( \delta \). The automaton \( B_1 \) cannot leave the state \( q^* \).

Thus we obtain the following lemma.

**LEMMA 1.** The automaton \( B_1 (1 \leq i \leq t) \) transforms a superword containing the letter \( \delta \) into a \( \delta \) superword.

The validity of the following lemma is easy to see.

**LEMMA 2.** An automaton \( B_1 (1 \leq i \leq t) \) with an initial state \( q_0 \) transforms the code of the word \( R \) in the alphabet \( A_1 \) either into the code of the word obtained from \( R \) by using the production \((P_1)\), or into a \( \delta \) superword.

Let us note that we have the first case if the word \( R \) begins with the word \( P_1 \).

Any automaton \( B_1 (1 \leq i \leq t) \) will be denoted by \( B_s \).

By induction on the length of a \((B_0, sB)\) chain we obtain from Lemmas 1 and 2 the following lemma.

**LEMMA 3.** An output superword of a \((B_0, sB)\) chain is either the code of a deducible word, or a \( \delta \) superword.

Let \( E_0 \) \( (E_1) \) be an automaton with two (one) inputs and one (two) state that realizes Sheffer's function (a unit delay) in the alphabet \( A_0 \) \( [7] \). It is well known that the basis \( \{E_0, E_1\} \) is complete.

Let \( B_R \) be an automaton (assigned to a nonempty word \( R \) in the alphabet \( A_1 \)) with one input and \( |R| + 2 \) states that is defined by the following system of instructions: