THEORY OF GROWING PYRAMIDAL NETS ON MATRIX STRUCTURES

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The paper considers the classification of growing pyramidal nets, describes the principles of their construction on matrix structures, and presents an appropriate algorithm.

The theory of growing pyramidal nets (GPN) was developed under the supervision of V. P. Gladun, Dr. Tech. Sci.

A pyramidal net is an acyclic directed graph such that no vertex has one incoming arc. Vertices without incoming arcs are called receptors, all other vertices are called conceptors.

Receptors in GPN correspond to the names of relations, properties, states, actions, objects, and classes of objects. Conceptors correspond to global descriptions of objects or situations, and also to intersections of these descriptions [1]. Formally, a GPN is defined by the triple \( S = (R, A, D) \), where \( R = \{r_1, \ldots, r_n\} \) is the finite set of receptors, \( A = \{a_1, \ldots, a_k\} \) is the finite set of conceptors, \( D = \{d_1, \ldots, d_s\} \) is the finite set of arcs connecting receptors and conceptors.

These sets define the GPN structure. GPN can be visualized as a directed graph with vertex set \( A \cup R \) and set of arcs \( D \).

The set of net vertices including the vertex \( a_i \) and all the vertices in the net graph from which connecting arcs lead to the vertex \( a_i \) is called the subset \( X_{a_i} \) of the vertex \( a_i \).

The set of net vertices to which arcs lead from the vertex \( a_i \) in the net graph is called the superset \( Z_{a_i} \) of the vertex \( a_i \).

The set of net vertices from the subset \( X_{a_i} \) of the vertex \( a_i \) connected directly with this vertex form the 0-subset \( X_{a_i}^0 \) of the vertex \( a_i \):
\[ X_{a_i}^0 \equiv X_{a_i} \cap A \cup R. \]

The set of net vertices from the superset \( Z_{a_i} \) of the vertex \( a_i \) connected directly with this vertex form the 0-superset \( Z_{a_i}^0 \) of the vertex \( a_i \):
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Note one characteristic feature of GPN: they are capable of changing their structure in order to adapt to descriptions of objects. This property is observed for pyramidal nets representing data whose mathematical model is a family of sets of symbols in a given alphabet \( \Sigma = \{k_i/i = 1, 2, 3, \ldots, n\} \) without any linear order relation. Pyramidal nets representing data of this type are called \( \alpha \)-pyramidal nets (\( \alpha \)-PN). Adaptation of \( \alpha \)-PN to descriptions of objects is accompanied by insertion of new vertices and arcs in the net and excitation of some group of receptors. The excitation process propagates through the net. A conceptor is excited if all the vertices of its 0-subset are excited [1].

The insertion of new vertices in $\alpha$-PN is described by the following rule: if the subset $F$ of the 0-subset of the vertex $a_i$ containing at least two vertices is excited when the object is perceived, the connections of the vertex $a_i$ with the vertices from the set $F$ are eliminated and a new vertex $a_{i+1}$ is adjoined to the net, whose inputs are connected with the inputs of all the vertices of the set $F$ and the output is connected with one of the inputs of $a_i$. The new vertex is in an excited state when inserted into the net [2].

The net vertices are thus connected by the presence of identical features in objects. For $\overline{F} = 2$, we achieve high differentiation of the net vertices by the set of identical features, but this increases the number of new intermediate vertices in the net, which in the final analysis increases the size of the GPN. It is thus relevant to extend the class of GPN by altering the lower bound on the number of excited vertices of the subset $F$.

GPN with $\overline{F} = h$, where $h = (3,4,5,\ldots,c)$, are classified as multiconnected $\alpha$-PN.

For the set $V = \{v_1, v_2, v_3, v_4\}$, where $v_1 = 111110$, $v_2 = 011110$, $v_3 = 111110$, $v_4 = 011110$, the directed graphs representing the $\alpha$-PN for $h = 2$, $h = 3$, and $h = 4$ are shown in Fig. 1.

A number of systems have been developed based on the premises of GPN theory [3]. The software systems Analizator, PPR1, PPR2 use list structures to represent GPN. However, matrix structures may be used for software modeling and also for efficient hardware support of GPN-based decision making systems.

We represent the GPN by a matrix in which the row indices are the indices of the concepts from the set $A$. A row of the matrix is the union of the vectors $M$ and $N$, where $M$ is the vector representing the description of the object whose elements are coded as binary digits and are numbered left to right according to the numbering of the receptors $r_n$ from the set $R$, and $N$ is the vector whose elements are coded as binary digits and are numbered left to right according to the numbering of the vertices of the set $A$, i.e.,

$$M = \{n_j; j \in 1, R\},$$

$$N = \{k_i; i \in 1, A\},$$

where $R$ and $A$ are the cardinalities of the sets $R$ and $A$, respectively,

$$n_j = \begin{cases} 0 & \text{if the feature corresponding to receptor } j \text{ is missing,} \\ 1 & \text{if the feature corresponding to receptor } j \text{ is present;} \end{cases}$$

$$k_i = \begin{cases} 0 & \text{if the vertex } a_i \in A \text{ corresponding to the } i\text{-th row of the matrix is not} \\ & \text{connected with the new vertex } a_{i+1} \in A, \\ 1 & \text{if the vertex } a_i \in A \text{ corresponding to the } i\text{-th row of the matrix is connected} \\ & \text{with the new vertex } a_{i+1} \in A. \end{cases}$$