DYNAMIC MULTI-BRANCH MODEL OF PRODUCTION (MODEL π)

Yu. P. Ivanilov and A. A. Petrov

We consider an open model of production consisting of N branches. It is assumed that each branch produces one product and that, in all the branches, there is the same duration of the production cycle, equal to unity in some accepted time unit. (As the unit of time one can adopt the year or, simply, the length of the production cycle itself.)

CONTROL SYSTEM AND CONSTRAINTS

We denote by \( x_i(t) \) the total production output in the \( i \)-th branch in year \( t \). By \( \xi_i(t) \) we denote the strength of the \( i \)-th branch at the beginning of year \( t \). We mean, by the strength of a branch, the maximal possible total output of the branch under the condition that its resources* over the course of a year will be retained at the same level they had at the beginning of the year. The strength of a branch is changed during a year by new resources entering into service (or dropping out) in this branch. We denote by \( \zeta_i(t) \) the strength of newly introduced resources during year \( t \). (If the resources decrease, then \( \zeta_i(t) < 0 \).)

The strength of the \( i \)-th branch at the beginning of the follow year is computed by equation

\[
\xi_i(t + 1) = \xi_i(t) + \zeta_i(t). \tag{1}
\]

The mean annual increase in strength \( \bar{\zeta_i(t)} \) in the \( i \)-th branch in year \( t \) will be somewhat less than \( \zeta_i(t) \)

\[
\bar{\zeta_i(t)} = \beta_i \zeta_i(t), \quad (\beta_i < 1). \tag{2}
\]

Here \( \beta_i \) is the coefficient converting the actual growth of strength over the year into the mean annual figure.

Consequently, in year \( t \) the total output of the \( i \)-th branch satisfies the inequality

\[
x_i(t) < \xi_i(t) + \bar{\zeta_i(t)}. \tag{3}
\]

New resources can be used for product output only after they have been completed. Moreover, after having been put into service, new resources must be mastered.

We denote by \( \delta_i(t) \) the additional strength of the \( i \)-th branch arising from resources whose formation began during year \( t \). The construction period in the \( i \)-th branch occupies \( l_i \) years.

We denote by \( \delta_i(t) \) the additional strength of the \( i \)-th branch arising from resources whose mastery began in year \( t \).

Let the mastery of resources in branch \( i \) begin \( m_i \) years after initiation of construction (\( m_i \leq l_i \)). Then,

\[
\delta_i(t) = \delta_i(t - m_i). \tag{4}
\]

This means that in year \( t \) strength has been mastered \( m_i \) years after construction of the relevant resources was begun.

*By resources we understand here the productive buildings and equipment.


© 1973 Consultants Bureau, a division of Plenum Publishing Corporation, 227 West 17th Street, New York, N.Y. 10011. All rights reserved. This article cannot be reproduced for any purpose whatsoever without permission of the publisher. A copy of this article is available from the publisher for $15.00.
The quantity $\zeta_i(t)$ is the accumulated sum of the strength mastered in year $t$. We denote the time of mastering the resources in the $i$-th branch by $n_i$. Then,

$$\zeta_i(t) = \sum_{s=0}^{n_i-1} \alpha_i(t, s) \delta_i(t-s) - \delta_i(t),$$

(5)

$\delta_i(t)$ is a given decrease in strength over the year due to deterioration of resources, while $\alpha_i(t, s)$ is the coefficient of annual increase in strength in branch $i$ during the $s$-th year of mastery.

Material outlays are necessary for the formation of resources for new strength. If we denote by $k_i(t)$ the material outlays on the $i$-th form of production in year $t$, then

$$k_i(t) = \sum_{j=1}^{N} \sum_{s=0}^{t-1} b_{ij}(t, s) \theta_j(t-s),$$

(6)

where $b_{ij}(t, s)$ is the coefficient of material outlay for the increase of unit strength during the $s$-th year of construction. It shows how much product of the $i$-th form must be expended in year $t$ on unit growth of strength of the $j$-th branch the construction for which began $s$ years ago.

Material outlays in year $t$ are implemented by changing the stores of product of the $i$-th branch, $q_i(t)$, and at the expense of that part of the production of the $i$-th branch, $z_i(t)$, applied to the accumulation of product

$$q_i(t+1) = q_i(t) + z_i(t) - k_i(t),$$

(7)

$q_i(t)$ is the reserve of product of the $i$-th form at the beginning of year $t$.

The growth of reserves, $q_i(t+1) - q_i(t)$ is introduced into the model as a balancing surplus. It can be unaggregated, it can be understood as the planned product which will be required in the future. In other problems, giving the structure of the reserves, this surplus can be understood as a supplement to some previously specified minimum requirement.

Finally, into the accumulated production of year $t$ goes that portion of the product of the $i$-th branch which remains after satisfaction of the current production requirements and end use.

$$z_i(t) = x_i(t) - \sum_{j=1}^{N} a_{ij}(t) x_j(t) - w_i(t).$$

(8)

Here, $a_{ij}(t)$ are coefficients of direct expenditures, i.e., quantity of product of the $i$-th form used to produce unit product of the $j$-th form.

To the relationships we have described it is necessary to add a number of constraints.

There is a constraint on the use of manpower resources in the productive sphere

$$\gamma(t) \leq \sum_{i=1}^{N} c_i(t) x_i(t) \leq \pi(t),$$

(9)

where $\pi(t)$ is manpower resources in year $t$, $c_i(t)$ is the coefficient of labor intensity, and $\gamma$ is the coefficient of admissible underexploitation of labor.

Further, it is necessary to impose the natural constraint

$$q_i(t) \geq 0,$$

(10)

which means that stocks cannot be negative.

If reserves cannot be used for material outlays to increase strength, it is then necessary to replace constraint (10) by the stronger

$$q_i(t+1) - q_i(t) \geq 0.$$  

(11)

Finally, it is possible to impose constraints on the strength under construction at the end of the planning period. Such strength defines the carryover to the following period. If the planning period encompasses $T$ years, then

$$\theta_i(T-s) \geq \theta_{is}^*, \quad s = 0, 1, \ldots, m_i - 1.$$  

(12)

The quantities $\theta_{is}^*$ must be specified.