A METHOD OF RECOGNITION LEARNING

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One of the classes of discriminant functions, applied in pattern recognition, is connected with the concept of committee [1]. A committee of a system of linear inequalities over a space \( \mathbb{R}^n \)

\[ (c_j, x) - a_j > 0 \]  

is a finite set of points of this space such that each inequality of system (1) is satisfied at more than half of the points of the set.

In [2] a definition is given of a committee of system (1) for \( a_j = 0 \) (\( j = 1, \ldots, m \)) and the condition of its existence is formulated: among the vectors \( c_j \) (\( j = 1, \ldots, m \)) none should be of opposed directions. In [3] a theorem was proved for the existence of a committee in the general case, and an algorithm for its construction is proposed and substantiated.

The committee concept can be applied to the following pattern recognition problem. Patterns \( \mathcal{A} \) and \( \mathcal{B} \), which it must learn to distinguish, are formally understood as the sets \( \mathcal{A} \subset \mathbb{R}^n \), \( \mathcal{B} \subset \mathbb{R}^n \). Information about the sets \( \mathcal{A} \) and \( \mathcal{B} \) consists in knowledge of certain of their finite subsets \( \mathcal{A} \subset \mathcal{A}, \mathcal{B} \subset \mathcal{B} \). A real separation function \( x \) is sought, in the space \( \mathbb{R}^n \) (in some concrete class of functions), with the property:

\[ \{x(a) | a \in \mathcal{A}\} \cap \{x(b) | b \in \mathcal{B}\} = \emptyset. \]

If the separation function \( x \) is sought in the class of linear functionals over \( \mathbb{R}^n \) (i.e., \( x(a) = (x, a) \) is the scalar product of a elements \( a, x \) of the space \( \mathbb{R}^n \)), then it is a solution of the system of inequalities:

\[ (x, a) > x_{a+1} (\forall a \in \mathcal{A}), \]
\[ (x, b) < x_{b+1} (\forall b \in \mathcal{B}). \]

where \( x = (x_1, \ldots, x_n) \), \( x_{n+1} \) is an unknown parameter.

If the system (2) is incompatible, but among the elements of the set \( \{(a, b)| a \in \mathcal{A}, b \in \mathcal{B}\} \) are no opposed vectors, then it is possible to find a committee \( (x^1 = (x^1_1, x_{1,n+1}), \ldots, x^q = (x^q_1, x_{q,n+1})) \) of this system. For \( q = 1 \) system (2) is compatible and sets \( \mathcal{A} \) and \( \mathcal{B} \) are separated by the linear functional \( (x^1, x) = x_1x_1 + \ldots + x_nx_n \), where

\[ A \subset \{x | (x^1, x) > x_{1,n+1}\}, \quad B \subset \{x | (x^1, x) < x_{1,n+1}\}. \]

To pattern \( \mathcal{A} \) are assigned all elements of the open half-space \( \{x | (x^1, x) > x_{1,n+1}\} \), and to pattern \( \mathcal{B} \) the elements of the half-space \( \{x | (x^1, x) < x_{1,n+1}\} \).

For \( q > 1 \), to determine to which set \( \mathcal{A} \) or \( \mathcal{B} \) an arbitrary element \( y \in \mathbb{R}^n \) is assigned, the scalar products

\[ (x^1, y) - x_{1,n+1}; \ldots; (x^q, y) - x_{q,n+1}. \]

are computed.

If the majority of these numbers is positive, then we assume that \( y \in \mathcal{A} \); if the majority is negative, then \( y \in \mathcal{B} \); in the third possible case \( y \in \mathbb{R}^n \setminus (\mathcal{A} \cup \mathcal{B}) \). Here, by definition of the committee, the elements of set \( \mathcal{A} \) will be assigned to \( \mathcal{A} \), the elements of set \( \mathcal{B} \) to \( \mathcal{B} \), i.e., the sets \( \mathcal{A} \) and \( \mathcal{B} \) are effectively separated.

Below we propose and substantiate a simple algorithm for finding a committee and its transformation in the addition of new inequalities (this corresponds to the presentation of new patterns in pattern recognition learning).

We put
\[
c_i = (a_n, a_{i+1}, \ldots, a_m),
\]
\[
f = (f_1, f_2, \ldots, f_n),
\]
\[
f' = (f_1', f_2', \ldots, f_n').
\]

For selected \( f, f' \in R^{n-2} \) the proposed algorithm for the construction and transformation of a committee of the homogeneous system
\[
(c_j, x) > 0 \quad (j = 1, \ldots, m)
\]
consists in the following.

1. The two-dimensional vectors
\[
d_i = \left( a_{i,a-1} + \sum_{k=1}^{n-2} a_{ik} f_k, a_i + \sum_{k=1}^{n-2} a_{ik} f_k \right)
\]
\[(j = 1, \ldots, m) \quad (4)\]
are brought into consideration.

2. A committee of the system of linear inequalities over the space \( R^2 \)
\[
(d_j, y) > 0 \quad (j = 1, \ldots, m). \quad (5)
\]
is found, consisting of solutions of its maximal compatible subsystems (mcs).

Let \( \{y^1, \ldots, x^q\} \) be a committee of system (5), \( y' = (x^t_{n-1}, x^t) \). Then it is easily seen that the set \( \{x^1, \ldots, x^q\} \), where \( x^t = (x_1, \ldots, x^n) \),
\[
x^t = f_1 x^t_{n-1} + f_2 x^t (k = 1, \ldots, n-2; i = 1, \ldots, q),
\]
\[
(6)
\]
is a committee of system (3).

3. If a new inequality
\[
(c_{m+1}, x) > 0
\]
is added to system (3), then we form the vector
\[
d_{m+1} = (a_{n+1,a-1} + \sum_{k=1}^{n-2} a_{m+1,k} f_k, a_{m+1} + \sum_{k=1}^{n-2} a_{m+1,k} f_k) = (d_{m+1,1};
\]
\[
d_{m+1,2}) \quad \text{and pass to the next step.}
\]

4. In the addition of the inequality
\[
(d_{m+1}, y) > 0
\]
to system (5), three cases are possible:

a) the set \( \{y^1, \ldots, x^q\} \), a committee of system (5), and the system
\[
(d_j, y) > 0 \quad (j = 1, \ldots, m + 1) \quad (7)
\]
is also a committee, so that in this case there is no need to transform it;

b) the set \( \{y^1, \ldots, x^q\} \) is not a committee of system (7), and for at least one element \( y' = y^i \) (\( i_0 \in \{1, q\} \)) a certain element \( z \) of the half-space
\[
|y'| (d_{m+1,1}, y) > 0
\]
belongs to the same mcs of system (3) as \( y^{i_0} \).