HOLOGRAPHIC EVALUATION OF STRAIN PATTERNS IN CYLINDRICAL SHELLS

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The surface curvature has to be considered in examining the strain in a cylindrical shell by holography, which is a difference from a planar object; this also complicates the quantitative interpretation of the pattern. In some studies we find either only qualitative analysis [2-4] or else no details of the quantitative method [5, 6] or analysis of the errors [7], the illustrations being restricted to the final result.

We have made a detailed interpretation of patterns via relationships derived for a cylindrical shell and have derived numerical values for the displacement and estimates for the error in reading the angular coordinates of interference fringes.

The material was a cylinder made of D16AT alloy (outside diameter 47 mm, inside diameter 46 mm, length 81 mm), which was loaded by internal air pressure \(0.43 \times 10^6 \text{ N/m}^2\). Figure 1 shows the interference pattern produced from a two-exposure hologram; the fringe pattern was analyzed by reference to the illumination and observation system (Fig. 2). Let \(S_i\) be the distance from the point source to some point on the object before deformation, and \(S_f\) the same after deformation; we see from Fig. 2 that

\[
S_i^2 = (h - R \sin \alpha)^2 + (l + R - R \cos \alpha)^2;
\]

\[
S_f^2 = (h - (R + \Delta R) \sin \alpha)^2 + (l + R - (R + \Delta R) \cos \alpha)^2.
\]

For symmetrical illumination and observation we get for the dark fringes that

\[
2\Delta S = 2(S_i - S_f) = \left(n + \frac{1}{2}\right) \lambda.
\]

We put \(S_i = S_f\) and subtract (1) from (2) to get

\[
S_i^2 - S_f^2 = (S_i - S_f) 2S_i = 2\Delta R (l + R) \cos \alpha + (h \sin \alpha - R)\lambda.
\]
We determine $S_1 - S_2$ from (4) and substitute into (3) to get
\[
\frac{2 \Delta R \left[ (l + R) \cos \alpha + (h \sin \alpha - R) \right]}{\sqrt{(l + R - R \cos \alpha)^2 + L^2 + (R \sin \alpha + h)^2}} = \left( n + \frac{1}{2} \right) \lambda. \tag{5}
\]

Table 1 gives numerical values for the increment $\Delta R$ in the shell radius in relation to the angular coordinate $\alpha$ of a fringe for the median section as derived from (5); the result derived from $\Delta R = \frac{pR^2}{Eh}$ is $7 \cdot 10^{-6}$ m, which may be compared with the tabulated values. The values of $\Delta \alpha$ show that for $\alpha$ close to the maximal value the error in determining $\Delta R$ is also maximal. The same conclusion is reached by relating the error $\Delta \alpha$ to $\alpha$ from (5) in accordance with Fig. 3. The value $\alpha_{\text{max}}$ is related to the angle of observation $\beta$. As $\beta$ decreases, $\alpha_{\text{max}}$ may increase up to a right angle, which thus reduces $\Delta R$ for large angular coordinates of the fringes.

We can use (5) with $L$ as variable (distance along the cylinder) to calculate the axial distribution of $\Delta R$ from the pattern of Fig. 1; Fig. 4 compares the theoretical and observed $\Delta R$.

It is difficult to measure $\Delta R$ near the fixing point of the cylinder by ordinary methods on account of the smallness of the displacements; however, holography allows one to determine small displacements, and the observed curve in Fig. 4 indicates a monotonic increase in $\Delta R$ where the theoretical calculation [1] gives a constant value. This is clear from Fig. 1, since four interference fringes are seen in a length of 27 mm on the cylinder reckoned from the median section. Also, the character of the fringes near the clamping point does not reveal the maximum indicated by the theory (Fig. 4).

It is also of interest to use (5) to determine the displacement near a localized force ($P = 65.5$ N) acting in the median section for a cylindrical shell made of D16AT (outside diameter 90 mm, inside diameter 86 mm, length 140 mm).

Figure 5a shows the interference pattern; Fig. 5b shows the deformation pattern as calculated from Fig. 5a (the axial coordinate of the section coincides with the point of application of $P$). However, in this mode of loading and recording it is impossible to record the displacement near the point of application of the force for the reasons stated above (Fig. 3); therefore, the mode of loading was altered so that the force vector lay in the same plane as the light source and point of observation.

Figure 6a shows the corresponding interference pattern; parts b and c of Fig. 6 show the displacements in the axial and radial directions calculated from this.