§1. Taylor transformations allow one to construct effective methods of solving linear and nonlinear differential equations [1-3]. It is, therefore, natural to consider the problem of feasibility of constructing computer-specialized procedures oriented toward automatic solution of Taylor equations. Several theoretical problems related to the creation of computational structures suitable for computers will be considered below.

§2. We recall the basic rules and equations of Taylor calculus. Let \( x = x(t) \) be a continuous function of argument \( t \), which can be represented by a convergent Taylor series on the interval \((0, H \leq \infty)\), and let \( X(k) \) be the Taylor transform of \( x(t) \), being a discrete function of the integer variable \( k = 0, 1, 2, \ldots, \infty \). The relation between the original \( x(t) \) and its transform \( X(k) \) is then defined as follows:

\[
X(k) = \frac{H^k}{k!} \sum_{l=0}^{\infty} \frac{\partial^l x(t)}{\partial t^l} \bigg|_{t=0} = \sum_{l=0}^{\infty} \left( \frac{t}{H} \right)^l X(k),
\]

where \( \equiv \) is the transform symbol from \( x(t) \) to \( X(k) \) and, inversely, from \( X(k) \) to \( x(t) \), \( H \) is a constant of dimension \( t \), and the argument \( k \) acquires the integer values \( k = 0, 1, 2, \ldots, \infty \).

The expression on the left of the symbol \( \equiv \) is the direct transform and that on the right is its inverse.

Denoting by \( x(t) \) and \( y(t) \) original functions, and by \( X(k) \) and \( Y(k) \) the corresponding T-transforms, the following relations hold:

\[
x(t) \pm y(t) \equiv X(k) \pm Y(k),
\]

\[
\lambda x(t) \equiv \lambda X(k),
\]

\[
x(t) y(t) \equiv X(k) Y(k),
\]

\[
\frac{x(t)}{y(t)} \equiv \frac{X(k)}{Y(k)},
\]

\[
\frac{d}{dt} x(t) \equiv D x(t) = \sum_{l=0}^{\infty} \frac{H}{k!} X(k+1) = \frac{H}{k!} \sum_{l=0}^{\infty} X(k+1),
\]

\[
\int x(t) dt = D^{-1} X(k) = \sum_{l=0}^{\infty} \frac{X(k)}{k!} + B x(k),
\]

\[
\varphi(k) = \begin{cases} 1, & k = 0, \\ 0, & k \neq 0, \end{cases}
\]

\[
x(0) = X(0), \quad x(H) = \sum_{k=0}^{\infty} X(k),
\]

\[
x(-H) = \sum_{k=0}^{\infty} (-1)^k X(k).
\]

Here \( \lambda \) is a nonvanishing constant, \( * \) and \( \equiv \) are symbols of the Taylor multiplication and division operations, \( D \) and \( D^{-1} \) are the symbols of Taylor differentiation and integration, \( B \) is a constant of integration determined by supplementary conditions, and \( \varphi(k) \) is the Taylor unity, being the T-transform of the function \( x(t) = 1 \).

Equations (9) indicate the following: 1) The initial values of the original and of its transform are always equal to each other, 2) the value \( x(H) \) of the original at \( t = H \) equals the sum of the discrete \( X(k) \), 3) the value...
\( x(\mathbb{H}) \) of the original at \( t = \mathbb{H} \) is equal to the sum of \( (-1)^k X(k) \) or, equivalently, to the sum of even discrete \( X(2k) \) minus the sum of odd discrete \( X(2k + 1) \).

We clarify the expressions above on simple examples.

**Example 1.** Find an analytic expression of a T-exponent, i.e., the discrete function being the transform of the exponential function \( \exp(\lambda t) = e^{\lambda t} \).

Since
\[
\frac{d}{dt} e^{\lambda t} = \lambda e^{\lambda t},
\]
by applying the direct transformation (1) to \( x(t) = e^{\lambda t} \) we obtain
\[
\exp(\lambda t) = \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} t^k,
\]
where \( e_\lambda(k) \) is the Taylor exponent.

**Example 2.** Find the T-transform of the hyperbolic sine \( \sinh(\lambda t) \).

Since
\[
\sinh t = \frac{e^t - e^{-t}}{2},
\]
it follows that
\[
\sinh(\lambda t) = \frac{e^{\lambda t} - e^{-\lambda t}}{2},
\]
where \( e_\lambda(k) \) is the transform of \( \sinh(\lambda t) \).

**Example 3.** Find the T-transform of the product of the functions \( x(t) = t \) and \( y(t) = e^{\lambda t} \).

Since \( t \leq H(k - 1) \) and \( e^{\lambda t} \leq \frac{(\lambda H)^k}{k!} \), we obtain by Eq. (4)
\[
T_t \sim \sum_{l=0}^{L} \frac{\lambda H(l - 1)}{(k - l)!} = \begin{cases} 0, & k = 0, \\ (\lambda H)^{k-1} \frac{1}{(k - l)!} & k \neq 0. \end{cases}
\]

**Example 4.** Find the T-transform of the ratio of functions \( \sin(\omega t) \) by \( \cos(\omega t) \).

It is easily seen that
\[
\sin(\omega t) = S_\omega(k) = e_{\omega}(k) \sin \frac{\pi k}{2},
\]
\[
\cos(\omega t) = C_\omega(k) = e_{\omega}(k) \cos \frac{\pi k}{2},
\]
where \( S_\omega(k) \), \( e_{\omega}(k) \) are the transforms of the functions \( \sin(\omega t) \) and \( \cos(\omega t) \).

Using Eq. (5) and taking into account that \( S_\omega(0) = 1 \), one obtains
\[
T_\omega(k) = \frac{S_\omega(k)}{C_\omega(k)} = \sum_{l=0}^{L} T_\omega(k - l) C_\omega(l) = \frac{(\omega H)^k}{k} \sin^2 \frac{\pi k}{2},
\]
where \( T_\omega(k) \) is the transform of the function \( \tan \omega t \).

**Example 5.** Find the T-transform of the derivative of the function \( \sin(\omega t) \) with respect to \( t \).

By Eq. (6) we obtain
\[
\frac{d}{dt} \sin(\omega t) = DS_\omega(k) = \frac{k + 1}{H} S_\omega(k + 1),
\]
but
\[
S_\omega(k + 1) = \frac{(\omega H)^{k+1}}{(k + 1)!} \sin \frac{\pi (k + 1)}{2} = \frac{(\omega H)^{k+1}}{(k + 1)!} \cos \frac{\pi k}{2},
\]

384