We study conditions for smooth and holomorphic linearization of germs of exterior differential 1-forms in a neighborhood of the origin. Let \( D(n) \) be the space of germs of all 1-forms \( \omega \in T^*\mathbb{R}^n (T^*\mathbb{C}^n) \), \( D_0(\omega) \) be the space of forms equal to zero at the origin. It follows from Darboux's theorem (cf. [1]) that one can linearize forms of \( D(n) \) in general position (in the smooth or analytic category). In [2] the linearizability of smooth forms in general position from \( D_0(n) \) is proved for even \( n \). In this note we prove an analog of Siegel's theorem (cf. [3]) for holomorphic forms from \( D_0(n) \) and the results of [2] are extended to the case of any dimension and are formulated as tests.

Let \( x = (x_1, \ldots, x_n) \) be an arbitrary coordinate system, \( \omega \equiv D_0(n) \), \( \omega = (A, dx) + \ldots + A \) be a matrix of order \( n \). Let us assume that \( \det A \neq 0 \). The eigenvalues of \( A^{-1}A^t \) (\( t \) is the symbol for the transpose) are invariants of \( \omega \), and we denote them by \( \lambda = (\lambda_1, \ldots, \lambda_n) \).

**Definition.** Let \( n = 2k \). The collection \( \tau(\omega) \) is said to be resonance, if either there exists an \( \eta = (\ldots, \eta) \) such that \( \lambda = (\lambda_1, \ldots, \lambda_n) \), or there exists an integral multiindex \( \eta = (\ldots, \eta) \) for which \( \eta = \lambda = (\lambda_1, \ldots, \lambda_n) \). Let \( n = 2k + 1 \). The collection \( \tau(\omega) \) is said to be resonance, if upon removing any number \( \eta(\omega) \) from it it becomes resonance, as a collection of even dimension.

**Remark.** For \( n = 2k + 1 \) the definition is connected with the fact that there always exists an \( \eta = (\ldots, \eta) : \lambda = (\lambda_1, \ldots, \lambda_n) \).

We denote by \( M(n) \) the set of forms in general position in \( D(n) \), singled out by the Darboux condition \( (n = 2k \Rightarrow \text{rank} \omega = n; n = 2k + 1 \Rightarrow \text{rank} \omega = n - 1, (\omega \wedge \omega)^{\theta} \neq 0) \). By \( M_0(n) \) we denote the set in general position in \( D_0(n) \), singled out by the conditions:

a) the linear approximation of a form \( \omega \equiv M_0(n) \) is nondegenerate;

b) the collection \( \tau(\omega) \) for \( \omega \equiv M_0(n) \) is not resonance.

The following theorem gives a test for 1-definiteness of smooth forms (1-definiteness is the property of linear approximation under which linearizability holds independent of the nonlinear part).

**Theorem 1.** In order that a smooth form \( \omega \equiv D(n) \) be 1-definite, it is necessary and sufficient that it belongs either to \( M(n) \) or \( M_0(n) \).

**Remark.** We give another definition of the set \( M_0(n) \), which does not require introducing coordinates. Let \( T_0, T_0^n \) be the tangent and cotangent spaces at \( \theta = \mathbb{R}^n \). For each form \( \omega \equiv D_0(n) \) we consider operators \( B_1(\omega) \) and \( B_2(\omega) \) from \( T_0 \) to \( T_0^n : B_1(\omega) \omega = d(\omega(X)) \) (linear approximation of the form \( \omega \); \( B_2(\omega) X = (d\omega(X)) \) \( \theta \). The operator \( B(\omega) = (B_1(\omega)) B_2(\omega) \) is an invariant of \( \omega \). The operator \( B(\omega) \) obtained from \( B(\omega) \) by factorization by the kernel of the form \( d\omega \), i.e., by the kernel of \( B_2(\omega) \), is also an invariant. The set \( M_0(n) \) consists of those and only those forms from \( D_0(n) \) for which the following conditions hold:

1. The linear approximation \( \text{the operator } B_1(\omega) \) is not degenerate.

2. The dimension of the kernel of \( B_2(\omega) \) does not exceed 1.

3. For the operator \( B^{-1}(\omega) \) the Poincare conditions hold (cf. [3]).

Now we proceed to the case of the analytic category.

**Definition.** Let \( n = 2k \). The collection \( \tau(\omega) \) has type \((C, \theta)\) if for any multiindex \( \alpha \), \( |\alpha| > 3 \) one has \( |\alpha| > C |\alpha|^{1-\theta} \). Let \( n = 2k + 1 \). The collection \( \tau(\omega) \) has type \((C, \theta)\) if there exists a \( \tau = (\ldots, \tau) \) such that upon removing from \( \tau(\omega) \) the number \( \tau_1(\omega) = 1 \) the collection obtained has type \((C, \theta)\) as a collection of even dimension.
The following theorem is an analog of Siegel's theorem.

**THEOREM 2.** In order that the holomorphic form \( \omega \equiv D(n) \) be 1-definite, it is necessary and sufficient that it belong to the set \( M(n) \subset D(n) \).

2. In order that the holomorphic form \( \omega \equiv D_0(n) \) be 1-definite, it is sufficient that the collection \( \tau(\omega) \) have type \((C, \theta)\) for some \( C > 0, \theta > 0 \).

We give the scheme of the proofs of Theorems 1 and 2. We start with Theorem 1. If \( \omega \equiv M(n) \) then the sufficiency follows from Darboux's theorem (cf. [1]), if \( \omega \equiv M_0(n), n = 2k \) then from [2] (the corresponding result in [2] is formulated in different terms). Let \( n = 2k + 1, \omega \equiv M(n) \). According to Darboux's theorem on 2-forms, \( \omega \) can be reduced to the form \( xy + dh(x, y, z) \), where \( x, y \in \mathbb{R}^k, z \in \mathbb{R} \). Then the collection \( \tau(\omega) \) not be resonance, one can conclude that \( \partial^2 H(0)/\partial z^2 \neq 0 \). Then changing the \( z \) coordinate, the form can be reduced to the form \( \tilde{\omega} = \pm zd\bar{z} + \omega_0 \), where \( \omega_0 = xy + dh(x, y, 0) \). The sufficiency in Theorem 1 now follows from the fact that the 1-definiteness of \( \tilde{\omega} \) is equivalent to the 1-definiteness of \( \omega_1 \) and the form \( \omega_1 \) as a form in \( \mathbb{R}^2 \) belongs to \( M_0(2k) \).

The necessity of the conditions of Theorem 1 for the case \( \omega|\theta = 0 \) is proved in [4]. It is easy to prove the necessity also in the case \( \omega|\theta = 0, n = 2k \). In the case \( \omega|\theta = 0, n = 2k + 1 \) if the conditions of Theorem 1 do not hold, the form can be reduced either to the form \( zdz + \omega_1 \), where \( \omega_1 \equiv \mathbb{R}^2 \) is a form which is not 1-definite, or to the form \( \partial^2 H(0)/\partial z^2 = 0 \). In the latter case the form is not even finitely definite [5].

Theorem 2 is proved with the help of the method of accelerated convergence (cf. [3, p. 194]). The first assertion of Theorem 2 is a consequence of Darboux's theorem and the results of [4]. Let \( \omega|\theta = 0, n = 2k \),

\[
\omega = \langle Ax, dx \rangle + f(x, dx), \quad f = \langle f_1, \ldots, f_n \rangle.
\]

We seek a linearizing substitution in the form \( x + \xi(x), \xi = \langle \xi_1, \ldots, \xi_n \rangle \). For \( \xi \) we get the equation \( L_A \xi + R_A(\xi, f) = -f, \) where

\[
L_A \xi = A\xi + (\xi')^t Ax, \quad R_A(\xi, f) = f(x + \xi) - f(x) + (\xi')^t (A\xi + f(x + \xi)).
\]

It is easy to show that the operator \( (\xi, f) \rightarrow R_A(\xi, f) \) has order 2. Hence to prove Theorem 2 it suffices to prove that the operator carrying the right side of the homology equation \( \delta \rightarrow L_A \xi = h, \quad h = \langle h_1, \ldots, h_n \rangle \) into its solution has order 1. For this we give a method of solution of the homology equation. Let \( T \) be a nondegenerate matrix such that \( P = T^{-1}A^{-1}AT \) is a Jordan matrix. We restrict ourselves for simplicity of the calculations to the case when \( P \) is a diagonal matrix: \( P = \text{diag}(\tau_1(\omega), \ldots, \tau_n(\omega)) \). We make a linear change with the matrix \( T \). The matrix of the linear approximation of the form will have the form \( A_T = T^t AT \). We are justified in considering the homological equation \( L_A \xi = h \). We seek a solution \( \xi \) in the form \( \xi = A_T \psi \). For \( \psi \) we get the equation \( \psi + (\psi')^t (A_T)^{-1} A_T = h \). But \( (A_T)^{-1} A_T = P^{-1} = \text{diag}(\tau_1^{-1}(\omega), \ldots, \tau_n^{-1}(\omega)) \). Now we introduce the scalar function \( u = x_1\psi_1 + \ldots + x_n\psi_n \). The equation found can be rewritten in the form of the system

\[
\frac{\partial u}{\partial x_1} - (1 - \tau_1^{-1}(\omega)) \psi_1 = h_1, \quad \ldots, \quad \frac{\partial u}{\partial x_n} - (1 - \tau_n^{-1}(\omega)) \psi_n = h_n.
\]

From these equations we express \( \psi_i \) in terms of \( u \) and we consider that \( u = x_1\psi_1 + \ldots + x_n\psi_n \). Here we get the equation for \( u \):

\[
u = \frac{x_1}{1 - \tau_1^{-1}(\omega)} \left[ \frac{\partial}{\partial x_1} - h_1 \right] + \ldots + \frac{x_n}{1 - \tau_n^{-1}(\omega)} \left[ \frac{\partial}{\partial x_n} - h_n \right].
\]

(1) can easily be solved in formal series. We note also that for any \( \mu \equiv \{1, \ldots, n\} \) one can find a \( f \equiv \{1, \ldots, n\} \) \( \tau_i^{-1}(\omega) = \tau_j(\omega) \). With the help of the usual calculations for the method of accelerated convergence we get that the operator \( h \rightarrow \psi(h \rightarrow \mu \rightarrow \nu) \) has order 1.

The case \( n = 2k + 1 \) can be reduced to the case \( n = 2k \) with the help of the same arguments as in the proof of Theorem 1.

**LITERATURE CITED**