COROLLARY. Let, in the notation of Theorem 1, \( f: C \rightarrow W \setminus D \) be a holomorphic mapping of lower order \( \lambda \leq 2 \) and \( D = \sum_{j=1}^{q} D_j \). Then \( f \) is equal to a constant if \( q > 4n \).

Remark. Let \( L \rightarrow W \) be a line bundle. The notation \( D_j \in \|L\| \) means that \( D_j \subseteq |\otimes^n L| \) for some natural number \( n \). One can prove that the Corollary to Theorem 1 remains true if the condition \( D_j \in \|L\| \) is replaced by the condition \( D_j \in \|L\| \).

2. Combining this with a theorem of Green [3], we get the following result.

THEOREM 2. Let \( W \) be a compact complex manifold \( \text{dim}_C W = n \), \( L \rightarrow W \) be a positive line bundle, \( D_j \in \|L\| \) be smooth divisors, and \( D = \sum_{j=1}^{q} D_j \) be a divisor with normal intersection. Then \( W \setminus D \) is a hyperbolic manifold if \( q > 4n \).

In particular, one has

THEOREM 3. Let \( \{V_j\}_{j=1}^{q} \) be a collection of smooth hypersurfaces of certain degrees in projective space \( \mathbb{P}_n(C) \), whose union has normal intersection. Then for \( q > 4n \), \( \mathbb{P}_n(C) \setminus \sum_{j=1}^{q} V_j \) is a hyperbolic manifold.

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LITERATURE CITED


BANACH SPACES, CONCAVE FUNCTIONS, AND INTERPOLATION OF LINEAR OPERATORS

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This paper is devoted to a study of interpolation spaces constructed from a pair of Banach spaces \( (A_0, A_1) \) and a concave, positively homogeneous function \( \varphi: \mathbb{R}_+^2 \rightarrow \mathbb{R}_+^1 \) of the first degree. This construction occurred for the first time implicitly in the work of Gagliardo [1] and was described explicitly by Gustavsson and Peetre [2] in a paper devoted to the proof of the fact that the space \( \varphi(L_{\varphi_0}, L_{\varphi_1}) \) (the notation is to be understood in the sense of the theory of ideal spaces [3]) is an interpolation space, where \( L_{\varphi_0} \) and \( L_{\varphi_1} \) are Orlicz spaces and \( \varphi \) satisfies a certain additional condition.

1. Let \( U \) be the set of all nonzero, nonnegative concave functions \( \varphi: \mathbb{R}_+^1 \rightarrow \mathbb{R}_+^1 \). We denote the set of all nonzero nonnegative functions \( \varphi: \mathbb{R}_+^1 \rightarrow \mathbb{R}_+^1 \) such that each \( \varphi \) is concave, continuous in the totality of its variables, and positively homogeneous of the first degree by \( W \). It is easy to see that if \( \varphi \in W \), then \( \varphi_0(x) = \varphi(1, x) \) and \( \varphi_1(x) = \varphi(x, 1) \) are elements of \( U \), and, conversely, if \( \varphi \in U \), then \( \varphi_0(x, y) = x\varphi(y/x) \) and \( \varphi_1(x, y) = y\varphi(x/y) \) are elements of \( W \).

For \( \varphi \in U \) we define \( \hat{\varphi} \) by

\[
\hat{\varphi}(y) = \inf_{x \in \mathbb{R}_+^1} \frac{1 + \varphi(x)}{\varphi(x)}.
\]  

(1)


The equality (1) defines an involution on U (this involution is closely related to the analogous involution of Lozanovskii [3]). The corresponding involution on W is defined by

\[ \phi(x, y) = \inf_{(x_0, x_1) \in \mathbb{R}^2} p(x, y, x_0, x_1). \]

Let \((A_0, A_1)\) be an interpolation pair [7], and suppose that \(\varphi \in W\). Following [2], we denote by \(\varphi^G(A_0, A_1)\) the set of all elements \(a \in A_0 + A_1\) for each of which there exists a sequence \(u = \{ u_i \} (i \in \mathbb{Z})\) of elements of \(A_0 \cap A_1\) such that \(a = \sum u_i\) (convergence in \(A_0 + A_1\)) and such that for any finite set \(F \subseteq \mathbb{Z}\) and any real sequence \(\xi = \{ \xi_i \} (i \in \mathbb{Z})\) with \(| \xi_i \| < 1\) we have that

\[ \left\| \sum_{i \in F} \xi_i u_i \right\|_{A_0} \leq c, \quad \left\| \sum_{i \in F} \xi_i u_i q_i (2^i) \right\|_{A_0} \leq c \]

(c does not depend on \(\xi\)). This space becomes a Banach space if we introduce the norm \(\| a \| = \inf c\). We note that \(A_0 \cap A_1\) is not, in general, dense in the space \(\varphi^G(A_0, A_1)\).

The space \(\varphi^G(A_0, A_1)\) is an interpolation space, and

\[ \| T \| \varphi^G(A_0, A_1) - \varphi^G(B_0, B_1) \| \leq \max \{ K_0, K_1 \}, \quad \max \{ K_0, K_1 \} = \| T \| A_1 \rightarrow B_1 \| . \]

In addition, we have the bound

\[ \| T \| \varphi^G (A_0, A_1) - \varphi^G (B_0, B_1) \| \leq cK_0M_0 (K_1K_0^{-1}). \]

where \(M_0\) denotes the dilatation function (see [8]) of \(\varphi_0(x)\).

Let \(X_0\) and \(X_1\) be Banach ideal spaces (see [9]), and suppose that \(\varphi \in W\). We will denote the construction of Lozanovskii [3] by \(\varphi(X_0, X_1)\). We denote the spaces of sequences \(x = \{ x_i \} (i \in \mathbb{Z})\) with norms \(x \| = \sum | x_i |\) and \(\| x \|_\infty = \sup_i | x_i |\) by \(l_1(\omega)\) and \(l_\infty(\omega)\), respectively.

**Lemma 1.** Let \( A_j = l_\infty (\omega_j) (j = 0, 1) \). Then the space \( \varphi^G (A_0, A_1) \) is isomorphic to the space \( \varphi (l_\infty (\omega_0), l_\infty (\omega_1)) \).

**Lemma 2.** Let \( A_j = l_1 (\omega_j) (j = 0, 1) \). Then the space \( \varphi^G (A_0, A_1) \) is isomorphic to the space \( \varphi (l_1 (\omega_0), l_1 (\omega_1)) \).

We can deduce the following theorem from Lemmas 1 and 2.

**Theorem 1.** Suppose that \(A_0 \cap A_1\) is dense in \(A_0, A_1\), and \(\varphi^G(A_0, A_1)\). Then the space \(\varphi^G(A_0, A_1)\) can be imbedded continuously in the space \(\varphi^G(A_0, A_1)^*\).

Suppose that \(\varphi, \varphi_0, \varphi_1 = W\) and that the numerical equality

\[ \varphi (x_0, x_1) = \varphi (\varphi_0 (x_0, x_1), \varphi_1 (x_0, x_1)) \]

is satisfied.

**Theorem 2.** The space \(\varphi^G (A_0, A_1)\) can be imbedded continuously in \(\varphi^G (\varphi_0^G (A_0, A_1), \varphi_1^G (A_0, A_1))\).

**Theorem 3.** Suppose that \(\varphi^G (A_0, A_1)^*\) reestablishes a norm in \(\varphi^G (A_0, A_1)\) and that \(A_0 \cap A_1\) is dense in \(A_0, A_1\). Then \(\varphi^G (\varphi_0^G (A_0, A_1), \varphi_1^G (A_0, A_1))\) can be imbedded continuously in \(\varphi^G (A_0, A_1)\).

Theorems 2 and 3 play the role of a reiteration theorem for the function \(\varphi^G\).

We note the connection of the spaces \(\varphi^G(A_0, A_1)\) with the spaces constructed in the best-known interpolation constructions.

The space \(\varphi^G (A_0, A_1)\) can be imbedded continuously in \((A_0, A_1)_{K, Y_0}\), where \(Y_0 = l_\infty (l_{\varphi_0} (\omega_0))\), and it contains \((A_0, A_1)_{K, Y_1}\), where \(Y_1 = l_1 (l_{\varphi_1} (\omega_0))\) (for definitions of these spaces, see [12]). If \(\varphi (x_0, x_1) = x_0^\theta x_1^{1-\theta} (0 < \theta < 1)\), then we have the continuous imbedding

\[ \varphi^G (A_0, A_1) \subset (A_0, A_1)_\theta, \]

where \([A_0, A_1]_\theta\) denotes the space obtained by Calderon's second method [7]. It is very interesting to clarify when the imbedding in (2) can be replaced by equality.