THE EPISTEMIC STRUCTURE OF A THEORY OF A GAME*

ABSTRACT. This paper is a contribution to the systematic study of alternative axiom-sets for theories of (normal-form, complete-information) games. It provides an introduction to epistemic logic, describes a formulation in epistemic logic of the structure of a theory of a game (the 'broad theory' of that game), and applies methods of epistemic logic to define strategies for dealing with two disturbing features of game theory, its hyperrationality assumptions and its indeterminacy. The analysis of these problems is conducted in terms of two principles which impregnate much game theory, Cleverness and Cloisteredness (the principles that players know respectively all, and only, the logical consequences of their assumed knowledge). Broad theories allow us to formulate and revise these principles despite their metatheoretical character. It is shown how Cleverness may be weakened by using logics which restrict the Rule of Epistemization, and Cloisteredness by using default logic or autoepistemic logic; the latter is used to characterize Nash equilibrium beliefs as parts of certain autoepistemic extensions of players' knowledge bases, but these particular extensions are rejected as ungrounded.

Keywords: epistemic logic, game theory, formal theory of rational play, logical omniscience, impossible world, default, autoepistemic logic, groundedness.

1. INTRODUCTION

This paper is a contribution to the 'meta-axiomatics' of game theory—that is, the systematic study of alternative axiom-sets for a theory of a game. It sets out to do three things: to provide an introduction to epistemic logic for decision theorists (Section 2); to give a formulation in epistemic logic of the structure of a theory of a game (Sections 3 and 4); and to apply methods of epistemic logic to define some possible strategies for dealing with two large problems for game theory: the hyperrationality, and the indeterminacy, of existing theory (Sections 5 and 6).

The task of normative game theory is to give and defend a specification of what rational players will do in the games they play. The heart of this task is epistemological. Once a player in a game has reasoned her way to beliefs, full or merely probabilistic, about what

her co-players will do, there is little more for her to do than to apply some more or less obvious principle of best reply. So game theory’s task essentially reduces to that of finding and defending a specification of players’ initial beliefs and inferential processes such that the latter lead them from the former to determinate beliefs about their co-player’s choices. Given any desired degree of determinacy of these beliefs, a tradeoff is forced on the theorist between endowments of initial beliefs and of inferential capabilities: the poorer she makes the former, the more powerful must be the latter; the weaker she makes the latter, the richer, must be the former. Traditional game theory makes the former subsistence-level, and the latter Herculean; but the might does not compensate for the poverty, and the conclusions are, notoriously, underdetermined.

Two principles impregnate the central research strategy of game theory, which correspond to the two components of the point on this tradeoff that has traditionally been chosen. I shall consider them in versions called Cleverness and Cloisteredness. Cleverness is a corollary of the assumption of ‘logical omniscience’, itself an epistemic dimension of hyperrationality: it is the principle that players know all logical consequences of their assumed knowledge. Cloisteredness is centrally implicated in indeterminacy: it is the principle that players know (about the game) only the logical consequences of their assumed knowledge. Notice that both principles are metatheoretical: they do not regard the game directly, but regard a feature of the game theorist’s theory of it, namely the ‘logical closure’ of the information attributed to the players in this theory’s assumptions. This metatheoreticity makes them tricky even to formulate, and a fortiori to appraise and amend if need be. To do these things scientifically one needs a framework in which a theory of a given game can itself be treated as a variable. This facility is provided by a logical formulation of a theory of a game presented below, a development of that of Bacharach (1987), to be called a broad theory of the game.

Section 2 describes epistemic logic. In Section 3, I discuss alternative ways, and propose a certain way, of defining ‘rationality’. In Section 4, I present the notion of a broad theory. A broad theory is a structure rich enough to allow one to formulate in it a range of radically different assumptions about what players know and how they know it.