We present conditions of asymptotic stability for the solution of a system of linear differential equations that depend on stochastic processes.

Let $\xi(t)$ and $\zeta(t)$ be independent Gaussian stochastic processes defined on $t \geq 0$ with values in $\mathbb{R}$ which have mean zero and correlation functions $NK(w(t-s))$ and $QK(w(t-s))$, respectively, where $N$ and $Q$ are $n \times n$ correlation matrices, $K(u)$ is a correlation function such that $K(0) = 1$, \[ \lim_{w \to \infty} V[w|R[K(w)]] = 0 \] for any $\varepsilon > 0$ independent of $w > 0$.

Consider a dynamic system whose behavior is described by the system of linear differential equations

$$
\frac{du_k^{(w)}(t)}{dt} = \sum_{i=1}^{m} A_{kj}(\xi(t)) u_i^{(w)}(t) + V w \sum_{i=1}^{m} B_{kj}(\zeta(t)) u_i^{(w)}(t)
$$

(1)

with nonstochastic initial conditions $u_k(0)$, $k = 1, \ldots, m$. Here $A_{kj}(x), B_{kj}(x)$, $k, j = 1, \ldots, m$, are the elements of the matrices $A(x), B(x), x \in \mathbb{R}^n$, $w$ is a parameter.

The aim of this study is to investigate the conditions of asymptotic stability, as $w \to \infty$, of the system of linear differential equations (1) dependent on stochastic parameters $\xi(t), \zeta(t)$. Stability of stochastic systems has been considered by various authors (see, e.g., [1, 2]). Let us state the basic results of the stability theory of stochastic systems in relation to our problem. We say that the trivial solution of the system of equations (1) is asymptotically Lyapunov stable with probability 1 as $w \to \infty$ if for any arbitrarily small $\varepsilon > 0$, $p > 0$ there is a number $\delta > 0$ such that $\|u(0)\| < \delta$ implies the inequality for the probability $\lim_{w \to \infty} \sup \{\|u^{(w)}(t)\| < \varepsilon\} > 1 - p$. For the trivial solution of the system of linear equations (1) to be asymptotically Lyapunov stable with probability 1, it is sufficient that a limiting Lyapunov function exists for system (1) as $w \to \infty$ such that the expectation of its total derivative with respect to time along the solutions of system (1) is negative.

Let $V(t, x), t \geq 0, x \in \mathbb{R}^m$, be a Lyapunov function twice differentiable with respect to $x$ and differentiable with respect to $t$ with a strict minimum at the origin, $V(t, 0) = 0$, and let $b_t(z), z \in \mathbb{R}^m$, be the (generalized) Fourier transform of the function $V(t, x)$:

$$
V(t, x) = \int_{\mathbb{R}^m} \exp \left\{ \sum_{k=1}^{m} x_k z_k \right\} b_t(z) \, dz.
$$

Assume that a function $\varphi_t(z)$ exists such that

$$
\lim_{w \to \infty} \left\| M^{(w)} \exp \left\{ \sum_{k=1}^{m} z_k u_k^{(w)}(t) \right\} - \varphi_t(z) \right\| = 0,
$$

(2)

where $M^{(w)}$ is the expectation operator over a distribution that depends on the parameter $w > 0$. Then the trivial solution of the system of equations (1) is asymptotically Lyapunov stable with probability 1 as $w \to \infty$ if

$$
\frac{d}{dt} \int_{\mathbb{R}^m} \varphi_t(z) b_t(z) \, dz < 0.
$$
Thus, in order to investigate the stability of the linear dynamic system (1), we need to establish the existence of a limiting function $\varphi_k(z)$ such that (2) holds. Let $E$ be a Banach space, $\Phi(t)$, $G(t, \lambda)$, $A^{(w)}(t, \lambda)$, $\lambda \in \mathbb{R}^n$, families of operator functions mapping $E$ to $E$ whose values are linear closed and in general unbounded operators with a definition domain everywhere dense in $E$. Consider the stochastic Cauchy problem for the equation

$$
\frac{dy^{(w)}}{dt} = \Phi(t) y^{(w)} + \int_{\mathbb{R}^n} \exp \left\{ i (\lambda, \xi(t)) \right\} G(t, \lambda) y^{(w)} d\lambda + 
$$

$$
+ \int_{\mathbb{R}^n} \exp \left\{ i (\lambda, \xi(t)) \right\} A^{(w)}(t, \lambda) y^{(w)} d\lambda, \quad y^{(w)}|_{t=0} = y_0,
$$

where $y_0$ is a point in the space $E$, round parentheses denote the scalar product in $\mathbb{R}^n$. Assume that the operator $A^{(w)}(t, \lambda)$ has an asymptotic expansion of the form $\frac{1}{V^w} A^{(w)}(t, \lambda) = A_d(t, \lambda) + \frac{1}{V^w} A_1(t, \lambda) + o \left( \frac{1}{V^w} \right)$, $w \to \infty$. Here $\sqrt{\omega}(1/\sqrt{w}) \to 0$ as $w \to \infty$ in the strong sense.

Consider the differential equation for the function $\varphi_k$:

$$
\frac{d\varphi_k}{dt} = \Phi(t) \varphi_k + \int_{\mathbb{R}^n} \exp \left\{ -\frac{1}{2} (Q, \varphi_k) \right\} A_d(t, \lambda) \varphi_k d\lambda + 
$$

$$
+ \int_{\mathbb{R}^n} \exp \left\{ -\frac{1}{2} (Q, \varphi_k) \right\} A_1(t, \lambda) \varphi_k d\lambda + 
$$

$$
+ \int_{\mathbb{R}^n} \int_{\mathbb{R}^n} \exp \left\{ -\frac{1}{2} (Q, \varphi_k) \right\} \exp \left\{ -K(u) (Q, \lambda) \right\} -1 \right\} \times 
$$

$$
\times \exp \left\{ -\frac{1}{2} (Q, \varphi_k) \right\} A_d(t, u) A_d(t, \lambda) \psi dud\lambda,
$$

$$
\varphi_k|_{t=0} = y_0.
$$

Applying the method of [3, 4], we can show that if for all $\varphi$ from $E$,

$$
\int_{\mathbb{R}^n} \exp \left\{ -\frac{1}{2} (Q, \varphi) \right\} A_d(t, \lambda) \varphi d\lambda = 0
$$

and if some fairly general conditions hold, then $\lim_{w \to \infty} \| M^{(w)} y^{(w)} \| - \varphi_k \| = 0$.

Let $g_{kj}(\lambda)$, $f_{kj}(\lambda)$, $k = 1, \ldots, m$, $\lambda \in \mathbb{R}^n$, be functions such that

$$
A_{kj}(x) = \int_{\mathbb{R}^n} e^{i(\lambda, x)} g_{kj}(\lambda) d\lambda,
$$

$$
B_{kj}(x) = \int_{\mathbb{R}^n} e^{i(\lambda, x)} f_{kj}(\lambda) d\lambda,
$$

and

$$
\int_{\mathbb{R}^n} \exp \left\{ -\frac{1}{2} (Q, \lambda) \right\} f_{kj}(\lambda) d\lambda = 0
$$

for all $k = 1, \ldots, m$. Let

$$
g^{(w)}(z) = \exp \left\{ i \sum_{k=1}^m z_k u^{(w)}_k(t) \right\},
$$