MODELING OF NONRIGOROUS REASONING IN FAMILIAR SITUATIONS

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Two axioms are proposed as the basis for a model of the decision-making process. The original information is expressed on the linguistic level, and the decisions are also made on this level. A technique is proposed for assessing the reliability of the decisions.

1. INTRODUCTION

The paper proposes an apparatus for modeling the decision-making process in situations that are familiar to the decision maker, in which the decision maker orients himself and acts with confidence. The model is useful when we need to generate some quick recommendations under a time constraint in an unfavorable psycho-physiological setting or when the operator is not highly qualified. Both the input information and the decisions are expressed in a fuzzy form using linguistic variables. A technique is proposed for assessing the reliability of the resulting decisions relative to the proposed model.

The apparatus is applicable to problems when the modeled decision maker has a good idea of the nature of all the factors and their effect on the outcome of the problem, possibly in qualitative, approximate terms. The construction of the model assumes that the decision maker has sufficient competence only with regard to the problem being considered: no special expertise is required in mathematical programming, nor is the decision maker expected to be able to formalize the problem by a system of criteria and to provide a comparative preference evaluation for several alternative solutions. The last requirement, often enforced in expert systems, may cause the experts psychological discomfort when faced with difficult to compare alternatives.

2. CONSTRUCTION OF THE MODEL

As a model reflecting the expert's view of the problem we take the digraph $G(V, U)$, whose structure should be established by an analyst familiar with formalization procedures. The vertex set of the graph $V = \{v_1, \ldots, v_m\} = \{q_1, \ldots, q_n, p_{n+1}, \ldots, p_m\}$ is partitioned into two nonintersecting subsets $V = P \cup Q$, $P \cap Q = \emptyset$, where $Q = \{q_1, \ldots, q_n\}$ are the output vertices and $P = \{p_{n+1}, \ldots, p_m\}$ are the input vertices. Each vertex $v_i \in V, i = 1, \ldots, m$, may take one of the $r$ values of a linguistic variable $(1, 2, \ldots, r)$, where $r$ is the same for all the vertices in the graph: this is a fundamental requirement of our method, as otherwise axiom A1 does not hold (see Sec. 3 on axiomatics).

The numerical value of a vertex corresponds to an element of a term set: thus, for $r = 3$, a vertex may take one of the values $\{v_i = 1 \text{ "small"}, 2 \text{ "medium"}, 3 \text{ "large"}\}$, for $r = 5$, $\{v_i = 1 \text{ "very small"}, 2 \text{ "small"}, 3 \text{ "medium"}, 4 \text{ "large"}, 5 \text{ "very large"}\}$, etc. As a rule, the expert can interpret without any difficulty the values of vertices of different nature. For instance, for the linguistic variable "frequency" ($r = 3$) we have (rare, frequent, very frequent), for "availability" (also $r = 3$) we have (shortage, sufficiency, excess), etc.

This linearly ordered representation is called S-fuzzy [1]: the term elements are linked by a transitive preference relation (better--worse, more--less), but there is no quantitative estimate of the extent to which one term is preferred to the other. If the expert cannot exactly identify one of the vertex values in a given $r$-partition, then he can indicate an interval of possible values (e.g., between "medium" and "very large"). One of the terms within that interval is chosen. Such "uncertainty" of the expert will influence the reliability of the decision (see Sec. 4).

The problem parameters identified by the expert are associated to the vertices of the graph. The output vertices $Q$ are the parameters whose values are to be determined in the decision process, i.e., the result is the assignment of appropriate values of the linguistic variables to the parameter vertices $q_i \in Q, i = 1, \ldots, n$. The input vertices $P$ are the parameters that in the expert's opinion influence the decision. Their values $p_j \in P, j = n + 1, \ldots, m$, should be known. In the simplest case $n = 1, m = 2, r = 2$, for instance, the output vertex $q_1$ is "decision about turning on the lamp," its value set is $(q_1 = 1 \text{ "turn on"}, q_1 = 2 \text{ "turn off"}); the input variable $p_2$ is "illumination" ($p_2 = 1 \text{ "dark"}, p_2 = 2 \text{ "light"}.$


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A directed edge \( u_{ij} \in U \) connects two vertices if in the expert's opinion the value of the vertex \( v_i \) influences the value of the vertex \( v_j \) (\( v_i \) is the visibility range and \( v_j \) is the speed of the automobile). This elementary connection will be denoted by \( u_{ij}(v_i \rightarrow v_j) \). To every edge the expert assigns a value \( |u_{ij}| \in [0, 1] \) which shows the degree of influence of the parameter \( v_i \) on \( v_j \) compared to other parameters that influence \( v_j \) (\( u_{ij} = 1 \) denotes very strong dependence, \( u_{ij} = 0.8 \) strong dependence, ..., \( u_{ij} = 0.2 \) weak influence, etc.).

A connection (an edge) is positive \( u_{ij} > 0 \) if higher values of \( v_i \) lead to higher values of \( v_j \); otherwise, \( u_{ij} < 0 \) (example: \( q_1 \) is speed, \( p_2 \) is visibility range, \( p_3 \) is difficulty of road conditions; then \( u_{q1} > 0 \) and \( u_{p3} < 0 \)).

Note that \( u_{ij}(v_i \rightarrow v_j) \) with \( u_{ij} < 0 \) always can be changed to an equivalent connection with a positive edge \( u_{ij} > 0 \) by replacing one of the vertices with its negation:

\[
\begin{align*}
\tilde{u}_{ij}(v_i \rightarrow v_j) &\iff \tilde{u}_{ij}(v_i \rightarrow v_j) \\
\tilde{u}_{ij}(v_i \rightarrow v_j) &\iff \tilde{u}_{ij}(v_i \rightarrow \tilde{v}_j).
\end{align*}
\]

In a verbal description, the negated vertices \( \tilde{v}_i \) are formed by adjoining the prefix "not" or by using an antonym for the vertex \( v_i \) (vulnerable—invulnerable, far—near). The value of \( \tilde{v}_i \) is defined by the negation operation on S-fuzzy sets [1]

\[
\tilde{v}_i = r + 1 - v_i.
\]

This operation has the properties of involution \( \tilde{\tilde{v}_i} = v_i \) and order inversion: if in situation \( x \) the value of the linguistic variable is \( v_i(x) \) and in situation \( y \) its value is \( v_i(y) \), where \( v_i(x) > v_i(y) \) (\( > \) is preference sign), then \( \tilde{v}_i(x) < \tilde{v}_i(y) \). Thus, for \( r = 5 \), the following values of the vertices \( p_i \) — "distant" and \( \tilde{p}_i \) — "near" are regarded as equivalent: \( p_i = 1 - \)