A NOTE ON THE DECIDABILITY OF DE FINETTI'S COHERENCE

ABSTRACT. To check the de Finetti coherence of a putative probability assigned to a class of events, we must know the possible combinations of truth values (constituents) of any finite class of events. Even for a very simple, finite, this can be impossible. In this case the notion of DF coherence cannot be applied to some or all the putative probabilities on this class of events.

Keywords: coherent probability, decidability, subjective probability, betting system.

1. INTRODUCTION

In his foundation of probability de Finetti (e.g. de Finetti, 1974) strongly advocates the use of a 'coherence criterion' to choose between putative probabilities, selecting those which can be considered 'reasonable'. The strength of this author's position comes (in his words) from the linkage of his definition of probability to an 'operational' criterion to check coherence.

As is well known, this point breaks down in the case of classes of events of non-finite cardinality. This is due to the need for the axiom of choice in order to apply de Finetti's definition to these cases. It is less well known that a similar, and perhaps more interesting, phenomenon may occur in the finite case. Briefly: given a putative probability on a finite class of events, the assessment of its coherence or incoherence can be undecided or undecidable (for the definition of these terms see Manin, 1978). This fact is a rather obvious application of a couple of famous results in mathematical logic. This notwithstanding, it seemed useful to point it out as an aid in evaluating the stance of de Finetti coherence, especially in view of the wealth of existence theorems produced in recent works about this subject.

This short communication contains an informal exposition of the subject, with some examples.
de Finetti considers probability assessments on a class $\mathcal{A}$ of events. An event is a proposition whose truth value can be ascertained in a well defined way (say with an experiment) at a well defined instant of time. The definition of coherence is needed to distinguish between 'admissible' and 'inadmissible' probability judgements. To be meaningful, this distinction has to be supported by an 'operational' criterion.

In de Finetti (1974), a putative probability $P$ on a class $\mathcal{A}$ of events is called coherent, or simply a probability, if:

$$\inf \sum \alpha_i I(A_i) \leq \sum \alpha_i P(A_i) \leq \sup \sum \alpha_i I(A_i).$$

Where $A_i$ is a member of any finite collection $\{A_i\}$ of events in $\mathcal{A}$, $\alpha_i$ is in any finite collection $\{\alpha_i\}$ of real numbers and $I(A_i)$ is 1 when $A_i$ is true and 0 if it is false.

The inf and the sup are taken w.r.t. all the possible combinations of truth values of the events in $\{A_i\}$. These possible combinations are called the constituents of $\{A_i\}$, and, in a sense, they correspond to the 'points' of a set where any $A_i$ in $\mathcal{A}$ can be represented as a point set.

The power and the charm of this definition stems from its interpretation in the light of the well known 'betting paradigm' of de Finetti (1974). The correct use of this definition, as of any other, depends on identifying its scope, i.e. the set of objects it applies to. In de Finetti (1974) and elsewhere (e.g. Regazzini, 1987) the following proposition is asserted:

PROPOSITION 2.1. Given any class $\mathcal{A}$ of events, it is possible to assign a coherent probability to it.

The following proposition is always (often tacitly) assumed, for instance in demonstrating 2.1:

PROPOSITION 2.2. Given any class $\mathcal{A}$ of events and any putative probability $P$ on it, we can always decide whether $P$ is or is not coherent.