easily be seen by taking \( c(r) = c\langle r \rangle = c \rightarrow 0 \) for one of the six directions characterizing the vectors \( \pm \mathbf{k} \), and setting \( c(r) = 0 \) for other directions. Then the second sum in (76) becomes equal to zero, while the first is equal to \( \epsilon \mathbf{j}^2 > 0 \). This proves that the form (76) is not nonpositive definite.

Physically, the result obtained implies the instability of a weakly inhomogeneous state with a bcc lattice in coordinate space. This agrees with Landau's assertion [6] regarding the impossibility of transitions of second kind from a homogeneous phase to an inhomogeneous phase with lattice \( L \) for which in the reciprocal lattice \( \hat{L} \) it is possible to find three vectors of "nearest neighbors" with the condition \( \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0 \).

In this situation we should seek as a basic weakly inhomogeneous state a state with lattice \( L \) for which in \( \hat{L} \) there are no triples of nearest neighbors with the condition \( \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0 \). Of lattices with cubic symmetry these are the lattices sc and fcc. The investigation of this problem will be published separately.

LITERATURE CITED


A DISCRETE VERSION OF THE LANDAU-LIFSHITS EQUATION

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A discrete version of the Landau-Lifshits equation from the theory of ferromagnetism is investigated within the framework of the method of the inverse-scattering problem. Variations of action-angle type are constructed, and the energy spectrum of the model is described. The procedure of "dressing" is used to obtain the simplest soliton solution.

An important direction in the investigation of completely integrable nonlinear equations is the construction of discrete analogs. In [2] and [3] a general method was developed which makes it possible to obtain lattice approximations preserving the property of complete integrability. Further investigations [4, 5] obtained by this method discrete models which corroborated the naturalness of the method.

The next step was taken in [1] where, following the discrete analogs of the nonlinear Schrödinger equation and sine-Gordon equation [3], a lattice analog of the famous Landau-Lifshits equation from the theory of ferromagnetism was constructed. In the present work we continue the investigation of the model described in [1] within the framework of the method of the inverse-scattering problem. As was expected, the properties of the discrete and continuous Landau-Lifshits models turned out to be very similar. This made it possible without any principle difficulties to construct exact solutions using only well-known methods given in [6] and [8]. The results obtained make it possible to hope for the usefulness of the model in applications, since none of the earlier known discrete analogs of the Landau-Lifshits equation was completely integrable.

The basis of this work was a thesis by the author written under the guidance of E. K. Sklyanin.
I. Description of the Model

We begin with the definition of the discrete analog of the Landau-Lifshits equation with periodic boundary conditions. Of course, here we only repeat the corresponding part of [1], somewhat modifying the sequence of the exposition and adding some details.

We consider a periodic, one-dimensional lattice of \( N \) nodes. To each node we assign a quadruplet of dynamical variables

\[ S^{(n)} = (S_{0}^{(n)}, S_{1}^{(n)}, S_{2}^{(n)}, S_{3}^{(n)}) \quad n = 1, 2, \ldots, N. \]

The Poisson brackets between the variables \( S_{\alpha}^{(n)} \) are quadratic [1]:

\[
\begin{align*}
\{ S_{\alpha}^{(n)}, S_{\beta}^{(n)} \} &= 2 J_{p}^{(n)} S_{p}^{(n)} S_{q}^{(n)} S_{nk}^{(n)} \\
\{ S_{\alpha}^{(n)}, S_{\beta}^{(n)} \} &= -2 J_{q}^{(n)} S_{p}^{(n)} S_{q}^{(n)} S_{nk}^{(n)} \\
J_{p}^{(n)} &= J_{p} - J_{q}^{(n)} \\
J_{1} &> J_{2} \geq J_{3}.
\end{align*}
\]

Here and below a triple of indices \( \alpha, \beta, \gamma \) denotes a cyclic permutation of \( (1, 2, 3) \). We note also that the sign of the constants \( J_{\alpha} \) has been changed as compared with [6] and [8] where \( J_{3} > J_{2} > J_{1} \).

On the variables \( S_{\alpha}^{(n)} \) we impose the constraints

\[
\sum_{\alpha=1}^{3} S_{\alpha}^{(n)} S_{\alpha}^{(n)} = k_{\alpha} \quad S_{0}^{(n)} + \sum_{\alpha=1}^{3} J_{\alpha}^{(n)} S_{\alpha}^{(n)} = k_{4}.
\]

The topology of the manifold defined by these constraints depends on the choice of the constants \( k_{\alpha} \) and \( k_{4} \). It is necessary to distinguish three cases:

a) \( K_{1} > K_{0}J_{4} \)

b) \( K_{0}J_{4} > K_{4} > K_{0}J_{2} \)

c) \( K_{0}J_{2} > K_{4} > K_{0}J_{3} \)

[In the case \( K_{0}J_{3} > K_{1} \) there are no real solutions of (3).] In this work we restrict our attention to the case \( K_{4} > K_{0}J_{3} \), since in this case it is possible to pass to the limit to the continuous Landau-Lifshits equation.

We define the \( L \)-operator of the model:

\[
L_{n}(u) = S_{0}^{(n)} + \sum_{\alpha=1}^{3} W_{\alpha}(u) S_{\alpha}^{(n)} S_{\alpha}^{(n)}.
\]

\( \sigma_{\alpha} \) are the Pauli matrices, \( W_{\alpha}(u) \) are expressed in terms of the Jacobi elliptic functions:

\[
\begin{align*}
W_{1}(u) &= \frac{k}{\text{sn}(u, k)} \quad W_{2}(u) = \frac{\text{dn}(u, k)}{\text{sn}(u, k)} \quad W_{3}(u) = \frac{\text{cn}(u, k)}{\text{sn}(u, k)} \\
W_{\alpha}(u) &= \sqrt{J_{\alpha}} \quad \text{for } \alpha = 1, 2, 3 \quad 0 < k < 1
\end{align*}
\]

We enumerate the symmetry properties of the operator \( L \) which we shall need below:

\[
L_{n}(u + 2k) = \sigma_{3} L_{n}(u) \sigma_{3}
\]