\[ |\varphi(t) - \exp(-t^2/2)| \leq \varepsilon^2 R^{2M} \]

for any values of \( t \) in the interval \([-\frac{AR}{2}, \frac{AR}{2}]\), and with

\[ M = \left( \ln \frac{1}{\varepsilon} \right) \left( \ln \ln \frac{1}{\varepsilon} \right) \]

As we can see from the proof of the theorem, it is easy to find a lower bound for the function \( \varepsilon_0(A, \delta) \), and we shall not dwell on this. Let us also note that it would be more accurate to consider instead of \( M \), its integer part \([M]\), but this likewise has hardly any effect on the results.

Let us illustrate the above theorem by some examples.

1. Let \( R = \exp \left( \ln \frac{1}{\varepsilon} \right) \left( \ln \ln \frac{1}{\varepsilon} \right) \). Then \( R^{2M} < \varepsilon^{-\eta} \) for any small number \( \eta > 0 \) if \( \varepsilon > 0 \) is small. Hence \( |\varphi(t) - \exp(-t^2/2)| < \varepsilon^\delta \) for any \( t \in \left( -\frac{AR}{2}, \frac{AR}{2} \right) \), where \( \delta \) can be taken as desired in the interval \((0, 1)\) if \( \varepsilon > 0 \) is sufficiently small.

2. Let \( R = \left( \ln \frac{1}{\varepsilon} \right)^{1/2}, 0 < \gamma < \frac{1}{2} \). In this case \( R^{2M} = \varepsilon^{-2\gamma} \), and therefore \( |\varphi(t) - \exp(-t^2/2)| < \varepsilon^{\delta - 2\gamma} \) for any assigned \( \delta \in (0, 1) \) and any \( t \in \left( -\frac{AR}{2}, \frac{AR}{2} \right) \).

In conclusion, let us note that the subject under discussion is encountered in certain problems involving positive-definite functions, for example, in the statistics of stationary processes with an analytic correlation function.

**LITERATURE CITED**


**RATE OF CONVERGENCE TO A STEADY STATE IN QUEUING SYSTEMS OF TYPE G\mid G\mid m\mid 0**

L. Seidl

Estimates that are uniform according to the number of servers are obtained for the rate of convergence to a steady state in queuing systems of type G\mid G\mid m\mid 0.

1. **Statement of the Problem**

We shall consider queuing systems of type G\mid G\mid m\mid 0, i.e., systems consisting of \( m \) (\( 1 \leq m \leq \infty \)) servers at which the calls arrive one by one and they are served by one of the free servers. If an incoming call does not find a free server, then it is not taken into consideration.

Suppose that the calls with numbers 0, 1, 2, ... arrive at the system at the instants \( t_0 = 0, t_1 = e_0, t_2 = e_0 + e_1, ... \), and that the service time of the \( n \)-th call is \( s_n \). We shall assume that \( P(e_j = 0) = P(s_j = 0) = 0 \) for any \( j \geq 0 \).

Let \( Y_j \) be the number of occupied servers at the instant of arrival of the call number \( j \) (\( j \geq 0 \)). For simplicity we shall assume that the call number \( j = 0 \) arrives at the system at the instant \( t_0 = 0 \) and that it finds the system empty, i.e., \( Y_0 = 0 \). The sequence of random variables (r.v.) \( X = (Y_0, Y_1, ...) \) is related to the defining sequence of the random vectors \( X = (X_0, X_1, ...) \), \( X_j = (e_j, s_j) \) as follows (see, for example, [1]).

**Transcribed from Problemy Ustoichivosti Stokhasticheskikh Modelei — Trudy Seminara, pp. 94-107, 1980.**
\[ Y_n = \sum_{k=0}^{n-1} I(s_k > e_k + \ldots + e_{n-1}) \delta(Y_k); \quad n = 1, 2, \ldots, \]

where \( I \) is the indicator of an event and \( \delta(Y_k) = 1(Y_k < m) \). In the notation for the sequence \( Y_n \) and the function \( \delta(\cdot) \) we omitted the dependence on the number \( m \) of servers. In the cases in which it is necessary to indicate this relationship, we shall use the notation \( Y_n^m, \delta^m(\cdot) \).

By \( \mathcal{S} \) we shall denote the set of all sorts of defining sequences (specified on the same probability space) that can be continued to narrow-sense stationary and metrically transitive sequences \( \{X_n: -\infty < n < \infty\} \) (see [2]). The continued sequences will be denoted in the same way as the original sequences, i.e., by \( X \). The subset of the set \( \mathcal{S} \) in which all the sequences \( X = \{X_n\} \) are independent will be denoted by \( \mathcal{S}_I \).

Let \( X \in \mathcal{S}, E_0 < \infty \), and let us introduce the following notation [under these conditions we denote by \( Q_k(x) \) a proper random variable]:

\[ Q_k(x) = \sum_{i=0}^{\infty} I(s_{k+i} > e_{k+i} + \ldots + e_{k+x}), \quad -\infty < k < \infty, \quad x > 0; \]

\[ A_k^L = \{Q_k(0) < L, Q_k(e_{k+1}) < L-1, \ldots, Q_k(e_{k+1} + \ldots + e_{k+L-1}) = 0\}, \]

\[ -\infty < k < \infty, \quad 1 \leq L \leq m. \]

We have the following assertion [1, Chap. 7, Theorem 1].

If \( X \in \mathcal{S}, P\left(\bigcup_{k=1}^{m} A_k^L \right) > 0 \), then for \( n \to \infty \) the distribution of the sequence of processes \( Y(n) = (Y_0+n, Y_1+n, \ldots) \) will converge to the distribution of a stationary process \( Y^* = (Y^*_0, Y^*_1, \ldots) \) such that \( P(Y^*_0 = m) < 1 \). Convergence is understood here in the following sense: There exist processes \( Z(n) = (Z^*_0, Z^*_1, \ldots) \) which are distributed in the same way as \( Y(n) \) and such that for \( n \to \infty \) we have

\[ P\left(\bigcup_{k=1}^{\infty} \{Y^*_k \neq Z^*_k\} \right) \leq P(Y^*_0 \neq Z^*_0) \to 0. \]

This paper is devoted to a study of estimates (uniform with respect to a set \( \mathcal{S}_I \in \mathcal{S} \) and to the number \( m \) of servers) of the rate of convergence of the sequence \( Z_0^n \) (the distribution functions \( F_{Z_0^n} \)) to the final random variable \( Y^*_0 \) (the distribution function \( F_{Y_0^*} \)) in the metrics \( \tau_1 \) and \( \sigma \), \( \kappa_1 \) defined below.

A method of construction of such estimates has been proposed by Borovkov [3]; with the aid of this method it was possible to obtain specific estimates for systems of type GI\|GI\|m and GI\|GI\|m\|0 by Borovkov [1], Akhmarov [4], and the author [5]. Let us note also the papers of Zolotarev [6] and of Kalashnikov [7], in which similar estimates have been obtained by another method.

2. System of Notations and Assumptions

By \( \mathfrak{S} \) let us denote a space of one-dimensional real r.v. defined on a fundamental probability space. Let us define the metrics in the space \( \mathfrak{S} \) that will be used below. Let \( Z \) and \( Z' \) be random variables that take their values in \( \mathbb{R} \). Let

\[ i(Z, Z') = EI(Z \neq Z'), \]

\[ \tau_1(Z, Z') = E |Z - Z'|, \]

\[ \sigma(Z, Z') = \sup \{ |P(Z \in A) - P(Z' \in A)| : A \in \mathfrak{S} \}, \]

where \( \mathfrak{S} \) is the system of all Borel sets in \( \mathbb{R} \);

\[ \kappa_1(Z, Z') = \int |P(Z < x) - P(Z' < x)| \, dx. \]

Let us note [1U] that any two r.v. \( Z, Z' \in \mathfrak{S} \) always satisfy the inequalities \( i(Z, Z') \geq \sigma(Z, Z'), \tau_1(Z, Z') \geq \kappa_1(Z, Z') \), whereas in the case of integer \( Z \) and \( Z' \) they satisfy also the inequalities \( \tau_1(Z, Z') \geq i(Z, Z') \) and \( \kappa_1(Z, Z') \geq \sigma(Z, Z') \). Therefore it suffices to obtain estimates of the rate of convergence in the metrics \( i \) and \( \tau_1 \) only.

In the following we shall select the defining sequences from a set \( \mathcal{S}_I \in \mathcal{S} \), on which we shall impose a collection of conditions such as those that follow below.