In the work, rays are considered which issue from the edge of a parabolic screen and after reflection from the convex side of the screen proceed in some fixed direction. Formulas are given for computing the trajectory and divergence of such rays. It is shown that for parabolic screens there exists a certain limit direction which has the character of a caustic boundary.

The possibility of distinguishing trajectories (optical paths) of the propagation of oscillations lies at the basis of the asymptotic approach in studying diffraction phenomena. From the convex side of a bounded screen which is part of a smooth convex curve in the plane there exist, in particular, optical paths having the character of rays issuing from the edges of the screen, being reflected several times from the surface of the screen, and passing to the point of observation. For a constant propagation speed \( c \) of the oscillations such a ray constitutes a broken line the adjacent links of which at the points of reflection from the screen satisfy the geometrical optics law of reflection. The calculation of the wave field propagating along such a ray consists in finding the ray and determining the divergence of the ray field in a neighborhood of it [1].

If the point of observation is removed to infinity, as is assumed, for example, in studying the direction diagram, then the condition of incidence of the last link of the ray at the point of observation is the condition that this link make a particular angle with some fixed direction.

In the general case the problem of finding a ray reflected several times from an arbitrary convex curve and issuing at a prescribed angle encounters definite difficulties. For example, for some curves several such curves possible, and apparently individual study in the case of each specific curve (possibly, some class of curves) is required.

In the present paper we consider a screen which is part of a parabola. Formulas are given for finding a ray reflected \( n \) times \((n = 1, 2, \ldots, N)\) and issuing at a prescribed angle. The question of uniqueness of the solution is studied. If a solution exists for a specific screen, then for exit angles \( \gamma \leq \pi/2 \) (the angle is computed from a line perpendicular to the axis of the parabola) it is unique; for angles \( \gamma > \pi/2 \) (these angles are possible only for \( n = 1 \)) two solutions are possible. The divergence of the ray field is computed in the second section of the paper.

It is possible that many of the results presented below could be proved on the basis of the geometric properties of confocal parabolas, proceeding from the fact that a ray inscribed in a parabola is tangent to some confocal parabola. An analytic method of investigation has been preferred, since the object of the work is to represent the solution in a form convenient for calculation on a computer.

1. Suppose that the screen is part of a parabola given by the formula \( y = x^2 \),* A ray issues from a point \( s_0 \) of the edge of the parabola; we shall assume that \( s_0 \) lies to the left of the axis of the parabola. We choose the positive direction on the ray (on the parabola) in the direction of increasing ray length (arc length of the parabola). If \( \alpha_i, \beta_i \) are the smallest angles between the oriented tangent to the parabola at the point \( s_i \) and adjoining links of the ray at this point (Fig. 1), then at points of reflection of the ray the following equalities hold:

*The more general case of the parabola \( y = ax^2 \) is equivalent to introducing the scale factor \( 1/a \) in the linear quantities.

We assume these angles to be positive.

Suppose that the link \( l_i \) issuing from the point \( s_i \) and the tangent to the parabola at this point make the respective angles \( \gamma_i \) and \( \beta_i \) with the x axis (positive angles are computed in a counterclockwise direction). These angles are obviously connected by the relations

\[
d_i = \gamma_i - \beta_{i-1}; \quad \beta_i = -\gamma_i + \gamma_i \quad (i=1,2,...,n)
\]

For the parabola \( y = x^2 \) there are the equalities

\[
tan \beta_i = x_i + x_i; \quad tan \gamma_i = 2x_i
\]

where \( x_i \) is the coordinate of the point of reflection \( s_i \).

We shall henceforth consider only screens for which rays reflected from some point "inside" the screen are impossible; with the present method of measuring angles at such a point, \( \alpha, \beta > \pi/2 \). It is found sufficient to require that at the first reflection the inequality \( \alpha_1 < \pi/2 \) is satisfied. This imposes a restriction on the choice of extreme points of the screen. From the expression for \( tan \alpha_i \) which can be obtained from relations (2) and (3) it follows that to satisfy this condition it suffices to assume \( |x_0| < \sqrt{2} \). This condition is also necessary if the screen is unbounded on the right or has edges which are symmetric relative to the axis of the parabola. If the screen is bounded also to the right of the point \( sp(x_F, y_F) \), then the coordinate \( x_F \) should also satisfy this condition.

Thus, assuming the conditions \( 0 < \alpha_i, \beta_i < \pi/2 \) are satisfied at points of reflection for the screens in question, it is possible to write the system

\[
tan \beta_i = tan \beta_l \quad (l=1,2,...,n)
\]

which is equivalent to system (1).

The first of the equalities (3) becomes meaningless if \( \gamma_i = \pi/2 \), which is possible only for the last link of the ray. Considering the last equation of system (4), it is easy to find that in this case the equality for the coordinates \( 1 + 4x_0x_l = 0 \) is satisfied. (This corresponds to the case where the penultimate link \( l_{n-1} \) of the ray passes through the focus of the parabola.) From what follows it will be clear that this is possible only for \( \gamma_i = \pi/2 \).

In all other cases relations (2) and (3) make it possible to represent system (4) in the form

\[
x_{i+1} = x_i - \frac{4x_i^2}{1 + 4x_0x_l} \quad (i=1,2,...,n)
\]

It is obvious that giving \( x_1 \) (\( x_0 \) is fixed) determines the entire sequence \( x_2, x_3, ..., x_n, x_{n+1} \).

Analysis of system (5) makes it possible to draw the following conclusions:

A) if the first point of reflection is chosen so that \( 1 + 4x_0x_1 > 0 \) (this corresponds to the case where the first link passes below the focus of the parabola), then \( -\pi/2 < \gamma_n < \pi/2 \);

B) the angle \( \gamma_n > \pi/2 \) is possible only under the condition \( 1 + 4x_0x_1 < 0 \) (this corresponds to passage of the first link above the focus of the parabola) and only in this case is \( n = 1 \).

We consider case A. From system (5) we obtain the equalities

\[
(t + 4x_i^2)(t + 4x_{i-1}x_i)(t + 4x_i x_{i+1}) = \bigg(1 + 4x_0x_l\bigg)^i \quad (i=1,2,...,n).
\]

Using them, we transform system (5) to the form

\[
\frac{x_{i+1} - x_i}{1 + 4x_0x_l} = \frac{x_{i-1} - x_0}{1 + 4x_0x_l} \quad (i=1,2,...,n),
\]

i.e., the quantities on the left are unchanged for all points of reflection. Verification shows that \( f(x_1) = (x_1 - x_0)/\sqrt{1 + 4x_0x_1} \) is a monotonically increasing function. Differentiating relation (6) with respect to \( x_1 \), we obtain