incident on the medium and leaving the medium is known, then there is the possibility in principle (and the method presented above of realizing it) of solving the following problems, and the solutions are unique in the sense indicated earlier:

a. knowing the scattering indicatrix and one of the functions \( \lambda(\tau), g(\tau) \), find \( I(\tau, \mu) \) and the second of the functions \( ^{\cdot}f(\tau), g(\tau) \).

b. knowing \( \lambda(\tau), g(\tau) \) and all the functions \( b_m(\tau), 1 \leq m \leq L, m \neq \ell \geq 1 \), find \( b_\ell(\tau) \) and \( I(\tau, \mu) \).

c. knowing \( \lambda(\tau), g(\tau) \) and the fact that the scattering indicatrix does not depend on \( \nu \), find \( I(\tau, \mu) \) and the scattering indicatrix.

If radiation is incident on the medium that varies so slowly that the transport can be considered stationary (the physical situation is the scattering of radiation in atmospheres of planets), then it is possible to determine \( \lambda(\tau), g(\tau) \) and the scattering indicatrix, \( b_m(\tau), 1 \leq m \leq L \).

If less information is known than in the versions described, the problems are, in general, unsolvable.

LITERATURE CITED

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THE WEAKLY NONREGULAR OPTICAL FIBER

A. P. Kachalov

In the paper the propagation of time-dependent electromagnetic waves in a weakly nonregular optical fiber of circular cross section is considered by means of the space-time ray method. A number of asymptotic formulas for such waves are obtained.

The extensive use of optical fibers in technology requires the study of their waveguide characteristics [1-3]. An especially complex problem is the study of the propagation of electromagnetic waves along a nonregular optical fiber. The technique of the space-time ray method has recently been developed [4] and can be applied to the calculation of a weakly nonregular optical fiber. This technique has previously been applied to study the propagation of waves in a weakly inhomogeneous, layered medium [5].

In this paper we shall study the propagation of time-dependent, modulated oscillations in a weakly nonregular optical fiber of circular cross section.

We consider a smooth curve \( l \) in \( \mathbb{R}^3 \) and a coordinate system \((x^i), i = 1, 2, 3\) in a neighborhood of this curve (see Fig. 1). The coordinates of the point of observation \( M \) are determined as follows: \( x^i = s \) is the arc length of the curve \( l \) from some fixed point \( M_0 \in l \) to the normal plane to \( l \) passing through the point \( M \), and \((x^2, x^3) = (r, d)\) are the polar coordinates of the point \( M \) in this normal plane with polar axis passing from a point on the curve \( l \) in the normal direction. The optical fiber consists of a medium with parameters \( \varepsilon, \mu \) filling the region \( r \leq \rho(s) \). Outside the optical fiber the medium has parameters \( \varepsilon_0, \mu_0 \).

In this coordinate system the contravariant components of the metric tensor have the form

Here $l_g = r^2 (1 - r K \cos \alpha)^2$ is the determinant of the matrix $g_{ik}$, $K$ is the curvature, and $\alpha$ is the torsion of the axis of the waveguide. We shall assume that the electric and magnetic permeabilities of the media $\varepsilon, \mu, \varepsilon_0, \mu_0$ depend only on the longitudinal coordinates $r, \alpha$.

To characterize the weak inhomogeneity of the optical fiber we introduce the large parameter $p$ and assume that

$$\xi = \xi (s'), \mu = \mu (s'), \varepsilon_0 = \varepsilon_0 (s'), \mu_0 = \mu_0 (s'),$$

$$g = g (s'), K = p^2 k (s'), \alpha = p^2 \alpha (s'),$$

where $s' = s/p$.

Our problem consists in solving the Maxwell equations inside the optical fiber and the Maxwell equations outside the optical fiber with conditions of exponential decay as $r \to \infty$ and the boundary conditions for $r = p$

$$E_\alpha = \varepsilon_0 E_\alpha, \quad E_\alpha = H_\alpha$$

$$E_S + \frac{2\varepsilon}{\varepsilon_0} E_\alpha = \varepsilon_S + \frac{2\varepsilon}{\varepsilon_0} E_\alpha,$$

$$H_S + \frac{2\mu}{\mu_0} H_\alpha = \mu_S + \frac{2\mu}{\mu_0} H_\alpha.$$  

Here and below $E, \varepsilon, H, H$ with lower indices are the covariant components of the electric and magnetic fields inside and outside the optical fiber.

Using formulas (2) and the expression for the contravariant components of the curl

$$\nabla \times V = \frac{1}{2 \sqrt{|g|}} \left( \frac{\partial V_i}{\partial x^j} - \frac{\partial V_j}{\partial x^i} \right) e^{ijk},$$

the first triplet of Eqs. (4) can be written in the form

$$\frac{\partial H_\alpha}{\partial \alpha} - \frac{\partial H_\alpha}{\partial x} = \frac{\varepsilon}{c (1 - r K \cos \alpha)} \left( \frac{\partial E_S}{\partial t} + \alpha \frac{\partial E_\alpha}{\partial t} \right),$$