LITERATURE CITED


STABILITY OF AN AUTONOMOUS DYNAMIC SYSTEM WITH FAST MARKOV SWITCHING

V. S. Korolyuk

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For an autonomous dynamic system with a fast Markov switching, sufficient conditions for asymptotic stability in the presence of a Lyapunov function which assures stability of a stationary averaged system are established.

Initially, stability of an input dynamic system satisfying the averaging condition — under the condition of stability of the averaged system — was established by Bogolyubov [1] (cf. also [2]). An analysis of stability of a differential-difference system with random perturbations over an averaged system is presented in paper [3].

In this paper, the martingale approach is used for investigation of stability of the initial stochastic system. This approach was developed in papers [4, 5] in the course of a study of systems with random perturbations under conditions of diffusion approximations.

1. Statement of the Problem. The original autonomous dynamic system with fast Markov switching is defined by a system of evolution differential equations

\[ dU^c(t)/dt = C(U^c(t), X^c(t)), U^c(0) = u_0, \]

where


Fast Markov switching is defined by a homogeneous, jumpwise, uniformly ergodic Markov process $X^c(t) := X(t/e)$ in a measurable phase space $(X, \mathcal{B})$. The generating operator $Q$ of the process $X(t)$ is defined by the kernel $Q(x, A), x \in X, A \in \mathcal{B}, q(x) := Q(x, X)$:

$$Q \varphi(x) := \int_X Q(x, dy)\varphi(y) - \varphi(x).$$

The stationary distribution $\pi(dx)$ is determined by the relation

$$\int_X \pi(dx) \varphi(x) = \int_X \pi(dy) Q(y, A), A \in \mathcal{B}.$$

The generating operator (2) of an ergodic Markov process generates the potentially bounded operator $R_\sigma$ [6]

$$R_\sigma \varphi(x) := \int_X R_\sigma(x, dy) \varphi(y),$$

possessing the following properties:

$$QR_\sigma \varphi(x) = R_\sigma Q \varphi(x) = \varphi(x)$$

for the functions $\varphi(x)$ satisfying the condition

$$\int_X \pi(dx) \varphi(x) = 0,$$

and in addition the orthogonality condition

$$\int_X \pi(dx) R_\sigma \varphi(x) = 0$$

is fulfilled.

It is well known (cf., e.g., [7]) that the corresponding stationary averaged system is defined by the deterministic evolution equation

$$\frac{dU(t)}{dt} = C(U(t)), U(0) = u_0,$$

in which the rate of the evolution is determined by the relation

$$C(u) := \int_X \pi(dx) C(u, x).$$

Our problem is to obtain sufficient conditions under which the existence of a Lyapunov function for the averaged system (7) assures the asymptotic stability of solutions of the initial system (1) for sufficiently small $\varepsilon > 0$.

In the author's paper [8], the condition announced is very far from the natural ones.

2. Statement of the Result. To simplify the notation of the vector and matrix operations, we shall introduce the notation: for a scalar product of a row-vector $c^* := (c_k; k = 1, d)$ by a column-vector $v := (v_k; k = 1, d)$:

$$c^* v := \sum_{k=1}^d c_k v_k;$$

for multiplication of a row-vector $c^*$ by a matrix $W := [w_{kr}; k, r = 1, d]$ on the left:

$$c^* W := \left(\sum_{k=1}^d c_k w_{kr}; r = 1, d\right).$$

In particular, $c^* W v := \sum_{k,r=1}^d c_k w_{kr} v_r$ is a scalar quantity and $c v^* := [c_k v_r; k, r = 1, d]$ is a matrix.