ACOUSTIC INSTABILITY OF MAGNETOGASDYNAMIC FLOW OF A DENSE PLASMA

I. M. Rutkevich

UDC 532.5.013.4:538.4

INTRODUCTION

Studies of the electrical conductivity of a dense nonideal metal-vapor plasma have shown that such media have possibilities for MGD energy conversion [1]. The determinative role in this connection is the sharp increase in the conductivity of the medium with increasing density: $(\partial \sigma / \partial \rho)_{T} > 0$. The rising dependence of $\sigma(\rho)_{T}$ has been established in experiments with mercury vapor [2-4] and cesium vapor [5].

The dependence of $\sigma$ on the thermodynamic parameters may lead to acoustic instability of flows in MGD channels. Acoustic instability has been studied in many papers (see [6-10], for example) for the plasmas used in open-cycle MGD generators and characterized by the conditions $(\partial \rho / \partial \rho)_{T} > 0$, $(\partial \sigma / \partial T)_{\rho} < 0$. In the present paper we investigate the acoustic instability of the flow in the channel of an MGD generator of a compressible medium under conditions typical for dense metal vapors: $(\partial \rho / \partial T)_{T} > 0$, $(\partial \rho / \partial T)_{\rho} < 0$. In addition to its distinctive conductivity, the medium to be considered differs from the plasma used in open-cycle generators and consists of the combustion products of an organic fuel by a weak Hall effect and relatively large values of the adiabatic exponent $\gamma$. The latter feature means that interaction between entropy and sound waves in the conducting medium must be taken into account.

The conditions for the loss of stability are determined for weakly nonuniform flows. It is shown that if the dependence $\sigma(\rho)_{T}$ rises sufficiently rapidly, steady flows become unstable starting from very small Mach numbers, corresponding to subsonic flows. Dispersion effects and types of instability are investigated. A range of parameters is discovered in which absolute instability is possible. A solution of the problem with initial conditions is constructed which illustrates the interaction (coupling) of an unstable sound wave and a stable entropy wave.

§ 1. We consider the unsteady flow of an isotropically conducting gas along the channel of an MGD generator; the two walls of the generator $y = \pm h(x)$ are solid electrodes, and the walls $z = \pm b$ perpendicular to the applied magnetic field $B(x)$ are insulators. Neglecting friction and heat exchange, the system of equations describing a quasi-one-dimensional flow can be expressed as follows in the customary notation:

$$\left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} \right) \rho + \frac{\rho}{h} \frac{\partial h}{\partial x} = 0$$

$$\rho \left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} \right) u + \frac{\partial p}{\partial x} = -\rho u B^2 (1-K)$$

$$\rho \left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} \right) S = c_v (\gamma-1) \rho u^2 B^2 (1-K)^{\gamma-1}$$

$$p = \rho RT, \quad S = c_v \ln \frac{T}{\rho^{\gamma-1}} + \text{const}, \quad \sigma = \sigma(\rho, T), \quad K = \frac{E}{uB}$$

The quantities appearing in (1.1) are averaged over the cross section of the channel. In the particular case of $h = \text{const}$, Eqs. (1.1) describe a strictly one-dimensional flow of a nonviscous and non-heat-conducting gas. The load coefficient $K$ for the generator mode of flow lies in the range $(0, 1)$. Equations (1.1) are valid provided the following conditions are satisfied:

$$Re_a = \mu_a u L \ll 1, \quad (w_* L/u) Re_a \ll 1$$

(1.2)

which enable the induced magnetic field and the rotational electric field to be neglected. Here $\omega_*$ is the characteristic frequency of the process and $L$ is the characteristic dimension of the nonuniformities (for example, the MGD interaction length in the case of a steady flow).

We are considering the region of thermodynamic parameters in which the nonideal character of the plasma shows up on its conductivity, but not on its equation of state \[1 \]. Experiments \[2, 11\] show that the ideal-gas equation of state adopted in (1.1) is a good approximation for \( \rho < 0.2 \rho_c, T > T_c \), where \( \rho_c \) and \( T_c \) are the parameters at the critical point.

Suppose small perturbations are impressed on a steady (background) flow. The parameters of the background flow, denoted below by the subscript \( \ast \), satisfy the steady-state system of equations obtained from (1.1) by setting \( \partial / \partial t = 0 \). Linearizing Eqs. (1.1) about the steady-state solution, taking the coefficients of the resulting equations as constants, and transforming to dimensionless variables via

\[
\begin{align*}
\tau = \frac{t}{t_0}, \quad \xi = \frac{x}{u_{0} t_0}, \quad t_s = \frac{\rho_0}{(1-K_0) \rho_0 B^2}, \\
\rho' = \frac{\rho}{\rho_0} - 1, \quad u' = \frac{u}{u_0} - 1, \quad S' = \frac{S-S_0}{c_v}
\end{align*}
\]

we arrive at the following equations:

\[
\begin{align*}
\frac{D\rho'}{Dt} + \frac{\partial u'}{\partial x} + C_4 \rho' + C_5 u' = & 0 \\
\frac{Du'}{Dt} + M^{-1} \frac{\partial u'}{\partial x} + \frac{1}{\gamma M} \frac{\partial S'}{\partial x} + C_6 u' + C_7 u' = & 0 \\
\frac{DS'}{Dt} + C_8 u' + C_9 u' = & 0 \\
\left( \frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \right), \quad M^{-1} = \frac{\gamma \rho_0}{\rho_0 u_0^2}
\end{align*}
\]

Here \( M \) is the Mach number in the unperturbed flow and the coefficients \( C_i \) are defined by the formulas

\[
C_1 = -u_{0} t_s (\ln \rho_0)_{\ast} \quad C_2 = -u_{0} t_s (\ln u_0)_{\ast} \quad C_3 = \gamma (\gamma - 1) (1-K_0)^{-1} + \lambda_s + u_{0} t_s [ (\ln u_0)_{\ast} + (\gamma - 1) M^{-1} (\ln \rho_0)_{\ast} ]
\]

\[
C_4 = (1-K_0)^{-1} + u_{0} t_s (\ln u_0)_{\ast} \quad C_5 = \gamma (\gamma - 1) (1-K_0)^{-1} + \lambda_s + u_{0} t_s M^{-1} (\ln \rho_0)_{\ast} 
\]

\[
C_6 = \gamma (\gamma - 1) (1-K_0) M', \quad C_7 = -\gamma (\gamma - 1) (1+K_0) M'^2 
\]

Here the subscript \( \ast \) denotes differentiation of the steady-state distributions with respect to the coordinate. The quantities \( \lambda_\rho \) and \( \lambda_s \) are defined by the expressions

\[
\begin{align*}
\lambda_\rho = \left( \frac{\partial \ln \sigma}{\partial \ln \rho} \right)_{\ast} = \left( \frac{\partial \ln \sigma}{\partial \ln \rho} \right)_{\ast} + (\gamma - 1) \left( \frac{\partial \ln \sigma}{\partial \ln T} \right)_{\ast} \\
\lambda_s = c_v \left( \frac{\partial \ln \sigma}{\partial \ln S} \right)_{\ast} = \left( \frac{\partial \ln \sigma}{\partial \ln T} \right)_{\ast}
\end{align*}
\]

In the linearization of (1.1), the fact that longitudinal perturbations do not lead to any change in the electric field is taken into account.

The assumption that the coefficients of the linearized system are constant is valid for the analysis of the stability of short waves whose spatial scale \( l \) satisfies the condition \( l \ll L_0 \), where \( L_0 \) is the length of variation of the background parameters. This assumption enables us to seek solutions of system (1.4) in the form of harmonic perturbations \( \exp [i(k \xi - \omega t)] \) for \( k \gg 1 \). Correct to terms of order \( k^{-1} \), the dispersion relation corresponding to system (1.4) has the form

\[
P(i) = \xi - M^{-2} \xi + \lambda_s \xi + \lambda_\rho \xi + u_{0} \xi = 0
\]

\[
a_s = -\gamma (\gamma - 1) (1+K_0) (\gamma - 1) M^{-1} + \lambda_s (\ln u_0)_{\ast} - (\ln \rho_0)_{\ast} \
\]

\[
a_t = (1-K_0)^{-1} + (\gamma - 1) (1-K_0) M' + u_{0} t_s [ (\ln u_0)_{\ast} + (\ln \rho_0)_{\ast} ]
\]

\[
a_z = -\lambda_\rho + (\gamma - 1) [ 1+K_0 (\gamma - 1) K_0 ] + u_{0} t_s [ (M^{-1} - 1) (\ln u_0)_{\ast} - (\ln \rho_0)_{\ast} ]
\]

The damping and the buildup of waves, and also their dispersion, are determined by the dissipative coefficients \( a_1 \) appearing in (1.7).

Dispersion relation (1.7) can be used to analyze both short harmonic waves and also localized perturbations (impulses) of width \( l \ll L_0 \). The quantity \( l \) also has a lower limit, since the effect of the thermal con-