STEADY AXISYMMETRIC FLOW WITH VORTICITY OF AN INVISCID FLUID IN MULTISTAGE TURBOMACHINES

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The direct problem of steady axisymmetric flow of a gas with vorticity through a multistage turbomachine is formulated precisely and a generalized solution is constructed by a variational--difference method. The turbomachine is represented schematically by an annular channel in which there are fixed ($\Omega_1$) and rotating ($\Omega_2$) three-dimensional cascades and channels free of them ($\Omega_0$).

In [1, 2], the boundaries $\Gamma_{ij}$ (i, j = 0, 1, 2) between the subregions of the different types are placed in front of and behind the edges of the blades, which, after averaging of the equations, makes it possible to smooth the solution and obtain continuity of the velocity vector in the case of flow past edges of finite thickness. However, this is valid only for small deviations of the flow from "shockless" entry, i.e., at small angles of attack.

With a view to extending the possibilities of the axisymmetric model of flow in turbomachines (with infinitely large number of blades and central interblade flow surface $S_0$, Fig. 1) with edges of arbitrary shape and for entry with shock, the region of rapid variation of the velocity vector at the entrance or exit from a cascade is contracted into the surface $\Gamma_{ij}$ of finite discontinuity of the velocity vector (vortex disk). The flow in each subregion $\Omega_i$ is described by a corresponding equation, and the coefficients of the highest derivatives of the equations at the boundaries (edges) have finite discontinuities. Mathematically correct and physically natural conditions for matching the solutions on these boundaries are obtained from the theory of generalized solutions of elliptic equations with discontinuous coefficients [3, 4]. There are "shock" losses of kinetic energy and inflections of the meridian streamlines at the edges.

In [5-9], attempts were made to construct a similar model of the flow, but only for shockless entry. In [10-14], shock entry and tangential inclination of the edges are considered, but these papers do not yet give a complete solution to the problem.

In this paper, we make more precise the axisymmetric flow model with finite discontinuities of the velocity vector at edges of arbitrary shape begun in [1, 10]. In contrast to the ones used earlier, such a model makes it possible to carry out calculations of an axisymmetric flow in not only the "design" regime and its neighborhood but also in a wider region of variable operation regimes of a multistage turbomachine. We present a method of constructing a generalized solution with the above matching conditions at the edges by means of a variational-difference method [15-17], and we give examples of calculations (realized with the BESM-3M and BESM-6 computers) for shock entry and tangential inclination of the blades with inflection of the meridian streamlines at the leading edge*; these correspond qualitatively to the experiment of [19].

1. Basic Equations

The system of equations for an axisymmetric steady flow with vorticity of a gas in a rotating cascade of a turbomachine can be written in the form [1, 14]

*The paper was presented at the Fourth All-Union Symposium on theoretical and applied mechanics [18].
\[
\frac{\partial w_r}{\partial r} - \frac{\partial w_z}{\partial z} + \frac{\text{ctg} \beta}{r} \frac{\partial (c_r)}{\partial r} + \frac{\text{tg} \delta}{r} \frac{\partial (c_r)}{\partial z} = \frac{1}{u_s} \left( \frac{\partial H^*}{\partial r} - \frac{T}{\rho_s} \frac{\partial S}{\partial z} \right) - \frac{f_y + f_z \text{tg} \delta}{u_s}, \quad f_\gamma = -\frac{T}{u_s} \frac{dS}{dt} \tag{1.1}
\]
\[
\frac{\partial (r \mu c_r w_r)}{\partial r} + \frac{\partial (r \mu c_r w_z)}{\partial z} = 0, \quad \frac{d}{dt} = w_r \frac{\partial}{\partial r} + w_z \frac{\partial}{\partial z} \tag{1.2}
\]
\[
\rho = \rho_{w*} \left( \frac{2H^* + u^2 - w^2}{2H^* + u_z^2} \right)^{\frac{1}{2}}, \quad \sigma = \exp \left( -\frac{\Delta S}{R} \right), \quad \frac{\Delta S}{R} = \chi \frac{RT}{\gamma - 1} \tag{1.3}
\]
\[
\frac{dH^*}{dt} = 0, \quad H^* = I + \frac{1}{2} (w^2 - u^2), \quad I = \frac{\gamma RT}{\gamma - 1} \tag{1.4}
\]
\[
w_r = w_r, \text{ctg} \beta + w_z, \text{tg} \delta, \quad c_r = c_r + u_r, \quad u = \omega r, \quad \chi = 1 - \frac{sN}{(2\pi r)}, \quad \rho_{w*} = \frac{p_{w*}}{RT_{w*}}, \quad \Delta S = S - S_1 \tag{1.5}
\]

Besides standard notation and notation explained by Fig. 1 and 2, we have here the following notation: \(c_r, c_u = c, c_z\) and \(w_r = c_r, w_u = c_u - u, w_z = c_z\) are the vectors of the absolute and relative velocities and their components in the cylindrical coordinate system \((r, \varphi, z)\); \(f\) is the vector of the frictional force tangent to the surface \(S_2\); \(\chi\) is the coefficient of restriction; \(\mu\) is the coefficient of blocking of the annular channel by the boundary layer [14]; \(N\) is the number of blades in the real cascade; and \(s = s(r, z)\) is their thickness in the circumferential direction. The asterisk is appended to the stagnation parameters, the subscript \(w\) to the stagnation parameters in the relative motion, and the subscript 1 to the parameters at the entrance to the cascade.

The system (1.1)-(1.5) is closed when the surface \(S_2\) is specified together with the field of the frictional forces \(f\) in terms of \(\sigma\) (or the loss coefficient \(\xi\)), since there are then five equations and as many unknowns: \(w, \rho, T\).

The equation of the flow surface \(S_2\) and the expression for the unit normal vector to it have the form
\[
\Phi = \Phi(r, z), \quad \Phi = c^2 \tag{1.6}
\]
\[
\nu = (itg \delta - j + kctg \beta)/\Delta \tag{1.7}
\]
\[
\Delta = (1 + tg^2 \delta + ctg^2 \beta)^n, \quad \text{tg} \delta = \frac{\nu_r}{\nu_u} = r \frac{\partial \Phi}{\partial r}, \quad \text{ctg} \beta = \frac{\nu_z}{\nu_u} = r \frac{\partial \Phi}{\partial z}
\]

The directions in which the angles \(\beta\) and \(\delta\) are measured are shown in Fig. 1.