Some general invariance relations are obtained for the integral diffusion fluxes of the reactant on the surface of one or several reacting particles of arbitrary shape in Stokes flow of a viscous incompressible fluid around the particles at large Péclet numbers. The case of irrotational flow is also considered.

1. Convective Diffusion to a Single Particle

We consider the two-dimensional (axisymmetric or plane) problem of steady convective diffusion to a reacting particle of arbitrary shape in a laminar flow of an incompressible fluid. We assume that on the surface of the particle there is complete transformation of the reactant dissolved in the fluid; its concentration far from the particle being constant. We also assume that the Péclet number \( P = \frac{aU}{D} \) is large; here, \( a \) is the characteristic dimension of the particle (radius of the sphere with equivalent volume), \( U \) is the characteristic flow velocity (at infinity), and \( D \) is the diffusion coefficient.

In the analysis, we use an orthogonal curvilinear coordinate system \( \xi, \eta, \lambda \) attached to the surface of the body. We assume that the coordinates \( \eta \) and \( \lambda \) are directed along the surface of the body, and \( \xi \) along the normal to it; the surface itself is specified by the fixed value \( \xi = \xi_0 \), and the flow field (and, therefore, the concentration field) does not depend on the coordinate \( \eta \) \( (\partial / \partial \eta = 0) \).

By virtue of the boundary conditions on the surface of the particle, the flow function near the particle can be represented in the general case in the form

\[
\xi \to \xi_0, \quad \psi(\xi, \eta) \to (\xi - \xi_0)^n f(\eta) \quad (g = g_{11} \xi g_{22} \eta g_{12}), \quad \nu_1 = -\sqrt{g_{11} \frac{\partial \psi}{\partial \xi}}, \quad \nu_2 = \sqrt{g_{22} \frac{\partial \psi}{\partial \eta}}, \quad \nu_3 = 0 \quad (1.1)
\]

Here, \( g_{11}, g_{22}, g_{12} \) are components of the metric tensor.

In concrete problems of the laminar viscous flow around particles with smooth surface, the parameter \( n \) in the relation (1.1) takes the value \( n = 1 \) for a drop or a bubble, and for solid particles \( n = 2 \) usually; in addition, there are some examples (transverse Stokes flow past a circular disk) for which \( n = 3 \) [1]. In the case of inviscid flow, for example, filtration problems, \( n = 1 \).

The zeros of the function \( f(\eta) \) in (1.1) determine the stagnation points (lines) on the surface of the body and determine the regions in which the sign of the flow function is constant. A stagnation point (or line) of the surface of the particle \( \xi = \xi_0 \) will be called a point of approach or recession if the normal component of the fluid velocity in its neighborhood is directed toward or away from the surface of the body, respectively. We denote the former by \( \eta_i^- \) and the latter by \( \eta_i^+ \) \( (i = 1, \ldots, m, \) Fig. 1\). Because of the conservation of mass, lines (points) of approach and recession must alternate. Moreover, in the axisymmetric case, for example, there are at least two isolated stagnation points lying on the symmetry axis, and in the plane case the number of stagnation points is always even. Figure 1 shows schematically the distribution of the fluid velocities near the surface of a body in the presence of stagnation points; the signs correspond to the sign of the flow function (1.1).

Note that in this paper we shall not consider the case of flow fields with closed

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streamlines surrounding the surface of the body; these can arise, for example, when a fluid moves in the gap between two rotating cylinders (i.e., when there are no stagnation points on the surface of the body). A fairly general analysis of such a situation is given in [2]. In addition, we shall not analyze flow fields with regions of a steady associated vortex of closed circulation next to the surface of a particle.

At large Péclet numbers and under the above conditions, the dimensionless total diffusion flux $I$ to the surface $S$ of the particle and the mean Sherwood number are determined by the expressions [1] ($0 < \lambda < \lambda_0$)

$$I = I(f) = \lambda_0 \nu^{-2} \Gamma^{-1} \psi(\nu) \prod_{i=1}^{n-1} \psi(\eta_{i+1}, \eta_i), \quad Sh = \frac{I}{S}$$

Consequence 1. It can be seen from the expression for the variable $t$ (1.3) and Eqs. (1.1) and (1.2) that when $\psi$ is replaced by $\psi^{-1}$, which corresponds to reversal of the direction of the fluid velocities (in the Stokes flow), the magnitude of the total diffusion flux does not change, i.e., $I(f) = I(-f)$. This result is not dependent on the type of exterior boundary conditions (at infinity) for the velocity field of the fluid, and can be readily generalized by means of the results of the works of [3, 4] to the case of arbitrary three-dimensional (Stokes) flow around bodies, this providing a generalization (in the limit $P \to \infty$) of the results of [5, 6] obtained for a translational [5] and a linear velocity field [6] of the fluid at infinity.

Consequence 2. If the shape of the body is symmetric with respect to the central line $n = \frac{1}{2} |n_1 - n_2|$, where $n_1$ and $n_m$ are the minimal and maximal roots of the equation $f(\eta) = 0$ determining isolated points of approach and recession, the total diffusion fluxes corresponding to the functions $f(\eta)$ and $\varphi(\eta) = f(|\eta_m - n| - \eta)$ will be equal, i.e., $I(f) = I(\varphi)$. This explains the results of [7, 8] obtained for a shear flow and a translational–shear flow. In particular, for [8] cases a) and c), and also b) and d) correspond to replacement of $\theta$ by $\pi - \theta$.

Consequence 3. Suppose the field of the flow past a drop or a bubble (for $n = 1$) is characterized by only two stagnation points on the surface, and the shape of the drop is symmetric with respect to the central line $n = \frac{1}{2} |n_1 - n_2|$. If at the same time the function $f = f(\eta)$ depends on the parameters $\alpha_1, \ldots, \alpha_n$ in such a way that the sum $f(|\eta_n - \eta| - \eta)$ does not depend on $\alpha_1, \ldots, \alpha_n$, then the total diffusion flux is also independent of $\alpha_1, \ldots, \alpha_n$.

A good illustration of this consequence is the translational–shear flow in cases a) and c) in [8], for which the total diffusion flux does not depend on the shear parameter $\omega_f$ and is represented by a straight line in Fig. 5 in [8].

Note that although the total diffusion flux in Consequences 1 and 2 is the same for the functions $f$ and $-f$ ($f$ and $\varphi$), the local diffusion fluxes on the surface of the particles will be different.

2. Convective Diffusion to Two (or Several) Particles

We consider here the problem of steady convective diffusion to two absorbing particles of arbitrary shape with fairly smooth surface in an incompressible Stokes flow. We shall assume that the particles are situated on the flow axis symmetrically with respect to the plane $z = 0$ and have only two stagnation points (situated on the flow axis), and that the dimensionless distance $l$ between them satisfies the inequality $l < O(P^{1/2})$ (here, as before, we take the characteristic length scale to be the radius of a sphere with the same volume as one of the particles).