THEORIES BETWEEN THEORIES: ASYMPTOTIC LIMITING INTERTHEORETIC RELATIONS

ABSTRACT. This paper addresses a relatively common "scientific" (as opposed to philosophical) conception of intertheoretic reduction between physical theories. This is the sense of reduction in which one (typically newer and more refined) theory is said to reduce to another (typically older and "coarser") theory in the limit as some small parameter tends to zero. Three examples of such reductions are discussed: First, the reduction of Special Relativity (SR) to Newtonian Mechanics (NM) as \( (v/c)^2 \to 0 \); second, the reduction of wave optics to geometrical optics as \( \lambda \to 0 \); and third, the reduction of Quantum Mechanics (QM) to Classical Mechanics (CM) as \( \hbar \to 0 \). I argue for the following two claims. First, the case of SR reducing to NM is an instance of a genuine reductive relationship while the latter two cases are not. The reason for this concerns the nature of the limiting relationships between the theory pairs. In the SR/NM case, it is possible to consider SR as a regular perturbation of NM; whereas in the cases of wave and geometrical optics and QM/CM, the perturbation problem is singular. The second claim I wish to support is that as a result of the singular nature of the limits between these theory pairs, it is reasonable to maintain that third theories exist describing the asymptotic limiting domains. In the optics case, such a theory has been called "catastrophe optics". In the QM/CM case, it is semiclassical mechanics. Aspects of both theories are discussed in some detail.

In an important paper (Nickles 1973) on reduction in physical theories Thomas Nickles argues that there are two distinct concepts of reduction appearing in the literature. The first is the usual "philosophical" sense which crudely speaking, depends on the deducibility of the reduced theory from the more fundamental reducing theory. It is often suggested that this sort of reduction leads to the explanation of the reduced by the reducing theory. But Nickles notes a second use of "reduction" – more often found in the writings of physicists than philosophers. In this sense the more fundamental theory is said to reduce to the less fundamental (typically superseded) theory in a limiting domain.

This paper is in part concerned with this second, physicists', sense of theory reduction; what Nickles called "reduction\textsubscript{2}". I agree that this is a legitimate use of the term "reduction", but I claim that its form and plausibility depends crucially on the nature of the relationship between
the two theories in the limiting domain. It has been said, for example, that special relativity (SR) reduces to Newtonian mechanics (NM) in the limiting domain where velocities are small compared with the speed of light. Similarly, it has been said that quantum mechanics (QM) reduces to classical mechanics (CM) in the limit as Planck’s constant $\hbar$ (or $\hbar = \hbar/2\pi$) approaches zero. A third case is the limiting relationship between wave optics and geometrical optics. My main motivation for studying intertheoretic reduction is not so much to try to make general claims about the nature of reduction as it is in understanding the particular and peculiar connections and correspondences between certain pairs of theories; particularly, between classical and quantum mechanics. It seems to me that contrary to popular belief, and in stark contrast to the case of SR/NM, there is a very important sense in which no reductive relationship obtains between members of the second two pairs of theories. The reason for the failure is interesting and leads to new physics and interesting philosophy which gets obscured if one accepts a claim of reduction at face value.

The paper begins with a discussion of some philosophical literature on intertheoretic reduction. I focus particularly on a discussion by Fritz Rohrlich concerning a form of reduction involving limiting relations between theories. A strong case can be made for the claim that NM reduces to SR along the lines of Rohrlich’s proposal. Following this, I consider the case of wave and geometrical optics and then turn to the quantum/classical case. The same conclusion is not forthcoming concerning these latter two cases.

A paradigm case where a limiting reduction rather straightforwardly does obtain is that of classical Newtonian particle mechanics and the special theory of relativity. In the limit where $(v/c)^2 \to 0$ SR reduces to NM. (I am going to drop the subscript on “reduction” as the kind of reduction discussed will be evident from the context.) We will consider this case first.

As Nickles says, “epitomizing [the intertheoretic reduction of SR to NM] is the reduction of the Einsteinian formula for momentum,

$$p = \frac{m_0v}{\sqrt{1 - \frac{v^2}{c^2}}}$$