Fig. 2 shows the results of the calculations of the angle of inclination of the shock wave for the case of flow past a cone with a half-opening angle $\omega = 10^\circ$ (1) exact solution; 2) linear theory; 3) Lighthill's solution; 4) proposed method. The results obtained are in good agreement with the solution found by the method of characteristics [4] for a solid of revolution with a contour $r = \varepsilon (2x - x^2)$ for $\varepsilon = 0.1428$, $M_\infty = 3$ (Fig. 3). In this figure we have used the following notation: 1) exact solution; 2) linear theory; 3) proposed method.

In the case of flow past solids of revolution at an angle of attack, the proposed method yielded good agreement with the exact solution and the experimental data (the experimental results were taken from the investigations of P. D. Mikhailov) both in the calculation of the coefficients of aerodynamic forces (Fig. 4) and in the determination of the distribution of pressure on the surface of the body (Fig. 5). Figure 4 shows the results for a solid of revolution with a contour $r = \varepsilon x^{2/4}$ with $M_\infty = 4$ (1) linear theory; 2) proposed method; 3) experiment. In Fig. 5 we show the data for a cone with $\omega = 15^\circ$ with $M_\infty = 3$ (1) exact solution; 2) linear theory; 3) proposed method) at three cross-sections corresponding in the cylindrical coordinate system to angles of $0^\circ$, $90^\circ$, and $270^\circ$.

In the processing of experimental data, we took account of corrections for the bottom resistance and the frictional resistance at zero angle of attack.

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LITERATURE CITED


DEPENDENCE OF POISEUILLE SLIP AND THERMAL CREEP ON THE LAW OF INTERACTION OF GAS MOLECULES WITH THE BOUNDARY SURFACE

P. E. Sustin and V. G. Chernyak

The problem of gas slip at a plane wall is solved on the basis of a linearized kinetic equation with a model collisional term (the S-model, which provides the correct value of the Prandtl number). Expressions are obtained for the Poiseuille slip and thermal creep which contain integrals of the scattering kernel over the velocity half-space. These expressions are made concrete for three particular cases corresponding to well-known models of the scattering kernel.

Sufficiently accurate solutions of the kinetic equations for the problem of gas slip in the Knudsen layer have now been obtained, e.g., [1-5]. However, the simplest models of fully diffuse or, in the best case, of specular — diffuse scattering of gas molecules from the boundary surface have been adopted as the boundary conditions. At the same time, it is interesting to investigate the effect of the law of the gas — surface interaction on the slip velocity. The present report is devoted to such an investigation.

The following approach to the statement of boundary problems of the dynamics of a rarefied gas has already become traditional: First, one chooses the kinetic equation, next one chooses some model of the interaction of the gas molecules with the surface over which they flow, and after this the boundary conditions...
are written down. Thus, the law of the gas—surface interaction is already contained in explicit form in the very statement of the problem. The defect of such an approach is obvious: Each time as information is accumulated on the laws of interaction of the gas molecules with the surface (i.e., as the boundary conditions are perfected) it is necessary to solve the kinetic equation anew for the calculation of macroscopic quantities.

In the present report the authors have departed from the traditional scheme. No concrete model of the gas—surface interaction is used in the statement of the boundary condition. The connection between the distribution functions for the molecules incident on and reflected from the wall is established by an integral expression through the scattering kernel [1]. Thus, the law of interaction of the gas with the surface over which it flows enters into the final expression for the slip velocity in a general form through the scattering kernel. The value of the result obtained is that it allows one to calculate the slip for any model of the interaction of a gas with a boundary surface without solving the kinetic equation anew each time in the process.

Let us consider the motion of a one-component monatomic gas, which fills the half-space \( x > 0 \), along the boundary surface \( x = 0 \) in the direction of the \( z \) axis.

Let the state of the gas be perturbed by a longitudinal temperature gradient \( dT/dz \) and let the motion be characterized by a macroscopic velocity \( U(x) \). If \( U(m/2kT)^{1/2} \ll 1 \) (\( m \) is the mass of a molecule, \( T \) is the gas temperature, and \( k \) is Boltzmann's constant) then the velocity distribution function of the molecules differs slightly from Maxwellian, i.e.,

\[
f(x, z, v) = f_0(z, v') \left[ 1 + h(x, v) \right]
\]

\[
f_0(z, v') = n(z) \left[ \frac{m}{2\pi kT(z)} \right]^{3/2} \exp \left[ -\frac{mv'^2}{2kT(z)} \right]
\]

where \( n \) is the numerical density of the molecules and \( v \) is the velocity of a molecule.

To simplify the calculations the original kinetic equation is chosen in the form of the S-model [6]. This model provides the correct value of the Prandtl number (in contrast to the BHC model) and consequently allows one to correctly describe the processes of heat and mass transfer at the same time. After linearization and reduction to dimensionless form the model kinetic equation is converted into an integrodifferential equation for the perturbation function \( h(x, v) \):

\[
c^2 \frac{\partial h}{\partial x} + c_t \left( c^2 - \frac{5}{2} \right) v = 2uc_t + Qc_t \left( c^2 - \frac{5}{2} \right) - h
\]

\[
Q = \frac{4}{15} \left( \frac{m}{2kT} \right)^{15/6} \rho^{-1} \int h c_t \left( c^2 - \frac{5}{2} \right) \exp(-c^2) dc
\]

\[
u = U \left( \frac{m}{2kT} \right)^{15/6} = \int h c_t \exp(-c^2) dc,
\]

\[
T_s = T(z=0), \quad \tau = \frac{1}{T} \frac{dT}{dz}, \quad z_0 = \frac{y}{2}, \quad s = \frac{y}{2}
\]

Here \( q_z \) is the heat-flux density in the direction of the \( z \) axis, \( p \) is the gas pressure, and \( l \) is the mean free path of the molecules, calculated from the coefficient of viscosity \( \eta = l/2 \rho u_0 (8kT_0/\pi m)^{1/2} \).

The boundary condition establishes the connection between the distribution functions of the molecules reflected from and incident on the wall [1]:

\[
f(x=0, v) = \int K(v', v)f(x=0, v')dv', \quad v_x > 0
\]

Here the scattering kernel \( K(v', v) \) is equal, with the accuracy of the factor \(-v_x/v_x\), to the density of the probability that a molecule incident on the wall with a velocity \( v_x \) is reflected from it with a velocity \( v \).

With allowance for the linearization of (1) and the principle of detailed equilibrium [2], Eq. (4) takes the form

\[
h^*(0, c) \exp(-c^2) = \int \exp(-c_x'^t) dc_x' \int \exp(-c_y'^t) dc_y' \times R(c', e)h^*(0, -c_z', c_x', c_y') dc_x' dc_y'
\]

\[
h = \frac{1}{2} \left[ (1+\text{sign} c_z) h^* + (1-\text{sign} c_z) h^- \right]
\]