The stability of gas flows produced by the motion of a flat piston or the decay of an arbitrary discontinuity is considered. The boundaries of the region (or regions) in which the development of perturbations is considered are planes (shock wave, contact discontinuity, piston, etc.) which move away from each other.

In Sec. 1 we consider the evolution of one-dimensional perturbations whose wave fronts are parallel to the boundaries of the region when at least one of the boundaries is the surface of a strong discontinuity which moves through the fluid particles. It is shown that flows of this class can exhibit a specific instability, which is reflected, in particular, in an instability in the position of the surface (or surfaces) of strong discontinuity. For a number of flows, criteria that take into account the stability of the position of the strong-discontinuity surfaces are given explicitly.

In Sec. 2 we study in the general case the behavior of gas-dynamic perturbations that can be represented as superpositions of arbitrarily oriented two-dimensional perturbations. Some simple arguments show that if the coefficients of reflection of the perturbations from the boundaries of the regions do not become infinite, then the development of perturbations that make an arbitrary finite angle with the boundaries at the initial time cannot lead to instability.

1. A characteristic feature of the considered flows is that the acoustic perturbations which arise in them can interact several times with the surface (or surfaces) of strong discontinuity in the flow. Such flows can exhibit an instability which is distinguished by the circumstance that the perturbations of the parameters of the motion and the state with discontinuity on the considered surfaces together with the perturbation of the velocity of the surface of discontinuity are damped with time whereas the amplitude of the displacement of the discontinuity surface from the position that it would occupy in the absence of perturbations increases. So can the amplitudes of the displacements of the fluid particles.

This comes about because the perturbations of the parameters on which the perturbation of the velocity of the discontinuity surface depends are not damped sufficiently rapidly. An instability of this kind can occur for discontinuity surfaces of various kinds — shock waves, detonation and combustion waves, strong discontinuities in magnetohydrodynamics, and so forth.

As an example, we consider the behavior of small perturbations in the gas between a fixed flat rigid wall and a parallel flat shock wave propagating through the gas away from the wall; we assume that in the unperturbed flow the parameters of the motion and the state of the gas in front and behind the shock wave are constant. We direct the x axis into the gas, taking the wall at the plane x = 0. With regard to the equations of state of the gas we shall assume that they satisfy the well-known restrictions of thermodynamic nature but are otherwise arbitrary. We shall consider below only the cases when \( M = U/a < 1 \), \( M_s = UV/Va_0 > 1 \), where U is the shock wave velocity, a is the velocity of sound, V is the specific volume in the undisturbed flow, and the subscript 0 is appended to the parameters ahead of the shock front. The initial conditions for the perturbations are assumed to be specified at \( t_0 \) and to be nonzero only in the region between the shock wave and the wall; further \( \zeta(y, z, t_0) \), where \( \zeta(y, z, t) \) is the displacement of the shock wave from the position (x = Ut) it would occupy in the absence of the perturbations.

We study the behavior of one-dimensional perturbations, assuming that the
perturbations $u'$ of the velocity, $p'$ of the pressure, and $S'$ of the entropy depend only
on $x$ and $t$, that $u'_x = u'_t = 0$, and that the perturbed flow is adiabatic. In this case,
the system of linearized equations for the perturbations has the invariants $I^\pm = p' \pm
au'/V$, where $I^\pm = f^\pm (t \mp x/a)$. The function $S'(x)$ in the perturbed region is determined from
the initial data and its values on the shock wave.

We note that after an interval of time $[t_0, t_0/a]$, where $a = (1 - M)/(1 + M)$, and in
the case of one-dimensional perturbations all initial perturbations that propagate with
the velocity of sound interact with the shock wave once, some of them having already
been reflected by the wall. For each $t \geq t_0$, we can find a natural number $n(t)$ such that
$a^{-n} \xi(t_0/a) = \xi(t_0/a)$. All $t$ satisfying this inequality for fixed $n$ belong to the time interval
during which the $n$-th reflection by the shock wave of all initial perturbations propagating
with the velocity of sound occurs. This interval of time is $(1 - a)t_0/a$ and, as can be seen, it increases with increasing $n$.

Using the properties of the invariants $I^+$ and $I^-$ and the boundary conditions which
must be satisfied in the linearized formulation by the perturbations on the shock wave
and the wall, we can readily establish that the behavior of the perturbations on the
shock wave can be described by the relations

$$
I^+(t) = K^a f^+(\alpha^a t), \quad I^-(t) = K^a f^+(\alpha^a t), \quad S'(t) = K^a S'(\alpha^a t), \quad U'(t) = K^a U'(\alpha^a t), \quad U'(t) = \frac{d\xi}{dt} = B I^+(t)
$$

$$
\zeta(t) = B \left( \sum_{n=1}^{n-1} \int_{t_0/a}^{t_0} \int_{t_0/a}^{t_0} I^+(\eta) d\eta + R^{-1} \int_{t_0/a}^{t_0} I^+(\eta) d\eta \right), \quad t_0 \leq \alpha^{-1} \xi(t_0/a), \quad m+1 = n(t), \quad R = Ka^{-1}
$$

(1.1)

Here, the subscript $H$ of a derivative means that it is taken along the direction of
the shock adiabatic curve.

As can be seen from (1.1) and was noted in [1], the flow is unstable for $|K| > 1$. For
$|K| < 1$, the perturbations $p'$, $u'_x$, $S'$, $U'$ on the shock wave, and $p'$, $u'_x$ in the entire
region behind the shock wave, are damped asymptotically. However, the asymptotic behavior
of the function $\zeta(t)$ for $|K| < 1$ depends on the ratio $K/a$. In formula (1.1) for $\zeta$
the terms collected together in the sum are due to the $(n-1)$-th reflection of the initial
acoustic perturbations by the shock wave. The coefficient of this sum is $\zeta(t_0/a)$. As a
function of $t$, this sum is a step function, and for $R \neq 1$ it is equal to $(R^n - 1)/(R -
1)$ and for $R = 1$ to $n - 1$. Note that the case $R = -1$ ($K = -a$) is not realized for $0 < M
< 1$. In the formula for $\zeta$ the value of the last term varies for fixed $n$ in the interval
from $0$ to $L |R|^{n-1}$, where $L$ is the maximal modulus of the function $\zeta(t)$ on the interval
$[t_0, t_0/a]$.

It follows from what we have said above that for $|R| < 1$ the function $\zeta(t)$ tends
with increasing $t$ to a limit equal to $\zeta(t_0/a)/(1 - R)$. It must be emphasized that for
$M < 1$ the shock wave can execute small undamped oscillations only when $R = 1$ and $\zeta(t_0/a)
= 0$. The last equation can be satisfied only for special initial data. For arbitrary
initial data, and, moreover, already at small $n$ in many cases, $|\zeta(t)|$ will increase.
Therefore, the case $K = a$, $M < 1$ is included in the instability criterion. For $|R| > 1$,
the amplitude of the displacement of the shock wave from the unperturbed position will
increase with increasing $n(t)$ even when $\zeta(t_0/a) = 0$.

Thus, if we are interested in not only the behavior of $p'$, $u'_x$, $S'$, $U'$ but also the
stability of the shock wave position, then with this broader understanding of the ex-
pression "flow stability" the considered flow will be stable in the framework of the linear theory for
$|K| < a$ and unstable for $|K| > a$ and $|K| = a$, $M < 1$.

The obtained criteria for a shock wave in gas can be formulated in the form of
restrictions on the slope of the shock adiabatic curve:

$$
|K| < a: f^+(\frac{dV}{dp})_H < 1 - 2M^2, \quad |K| > a, \quad K = a, \quad M > 1: f^+(\frac{dV}{dp})_H < -1; \quad f^-(\frac{dV}{dp})_H > 1 - 2M^2
$$

(1.2)