PROPAGATION OF AN ELECTROMAGNETIC WAVE IN A COAXIAL CABLE.

THE CASE OF A NARROW GAP

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A study is made of the propagation of a discontinuity of the electromagnetic field in a magnet in a coaxial cable. The shape of a discontinuity of the electromagnetic field propagating along a magnet from a voltage source is found. Under the assumption of a small gap between coaxial, infinitely conducting cylinders, the field parameters behind the discontinuity are determined.

Simple waves and discontinuities in media whose magnetic permeability is a function of the modulus of the magnetic field intensity were considered in [1]. The propagation of shock waves in coaxial cables filled with a ferrite with rectangular magnetization curve and methods of engineering calculations are discussed in [2, 3].

If a discontinuity of the electromagnetic variables propagates in an infinite layer of a nonconducting magnet in the absence of free charges, the field parameters behind and ahead of the discontinuity are related by [4]

\[ [B_\tau] = 0, \quad [D_n] = 0, \quad [E_\tau] = -\frac{1}{c} (\langle B \times u \rangle)_n, \quad [H_\tau] = -\frac{1}{c} (u \times D)_t. \]  

Here, \( n \) is the direction of the normal, \( \tau \) is the direction of the tangent to the discontinuity plane, \( u = un \) is the normal component of the velocity of propagation of the discontinuity through the magnet, and the square brackets indicate the difference between the values of the parameters behind and in front of the discontinuity.

It is obvious that such a discontinuity can propagate in a half-space filled with a magnet containing no electromagnetic field if an electric field \( E \) is applied instantaneously to the plane bounding this half-space and then maintained at a constant value; the electric field can always be assumed to be directed along the \( y \) axis: \( E = Ey \). In the region behind this discontinuity, the electric and magnetic fields are constant. The velocity of the discontinuity and the magnetic field behind it can be found from Eqs. (1) with allowance for the circumstance that the electric and magnetic fields in front of the discontinuity are zero, and the electric field behind the discontinuity is equal to the vector \( E \) applied to the plane bounding the half-space.

A similar wave will propagate in a layer of magnet between two infinitely conducting planes \( y = \pm h \) (Fig. 1), since the above solution satisfies a boundary condition on the conducting planes in the form of the vanishing of the tangential component of the electric field. On the parts of the planes \( y = \pm h \) bounding the disturbed region, a surface charge \( q \) appears. Through the known boundary condition, this charge can be represented by the formula \( q = \varepsilon Ey / 4\pi \) (where \( \varepsilon \) is the permittivity of the magnet). In what follows, without loss of generality we shall assume that \( \varepsilon = 1 \), and that there is no electric field outside the magnet for \( |y| > h \), since the planes \( y = \pm h \) are perfect conductors. The motion of such charges along the perfectly conducting surfaces corresponds to the presence of a surface current determined by the formula \( j = \varepsilon Ey / 4\pi \) (where \( v \) is the velocity at which the charges move along the surface). In its turn, the current \( j \)
produces in the magnet the magnetic field $B_z = \mu j/4\pi c$ behind the wave front.

Further, we shall consider the propagation of an electromagnetic wave between two infinitely conducting coaxial cylinders having a gap between them small compared with the radii of the cylinders and filled with a nonconducting magnet (magnetic permeability $\mu$ of the magnet, gap $\delta$ between the cylinders, and radius $r_0$ of the inner cylinder given).

If the cylinders are connected to a voltage source, then, as in the case with planes, an induced surface current will flow along the conducting surfaces of the cylinders and produce an electromagnetic field in the magnet. We shall seek the solution to the problem of the propagation of the electromagnetic field through the magnet under the assumption that there propagates through it a curved (because of the radial inhomogeneity of all the quantities) discontinuity, this being followed with the same velocity $v$ by a certain zone in which all the variables of the electromagnetic field are equalized a long way behind the discontinuity (as $x \to -\infty$) to the values $H_0 = C_1/r$ and $E_x = 0$, $E_r = C_2/r$ (where $C_1$ and $C_2$ are constants). We assume also that all the quantities depend only on the coordinates $x$ and $r$ and the time $t$, that the magnetic field everywhere within the magnet has component $H_z$, and that the electric field is perpendicular to it and has components $E_x$ and $E_r$. We determine the electromagnetic field behind the discontinuity and the shape of the discontinuity profile.

In the zone following the discontinuity, in which the parameters vary continuously, the relations between the electric and magnetic fields are described by Maxwell's equations, these taking in the cylindrical coordinate system the form

$$\frac{\partial H_x}{\partial x} - \frac{\partial E_r}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} (rH_z) = \frac{\partial E_x}{\partial t} - \frac{\partial E_r}{\partial r} = -\frac{\partial B_z}{\partial t}$$

We shall assume that the solution of the system (2) depends on the combination $x - vt$ of the variables and on the coordinate $r$. Since the radii of the cylinders are nearly equal, $r = r_0 + y$, where $r_0$ is the radius of the inner cylinder, $0 \leq y \leq \delta$, and $\delta \ll r_0$.

We represent the basic quantities in the form $H_z = H_0 + h$, $B = B(H_0 + h) = B_0 + B'h$, $E_r = E_0 + e_y$, $E_x = e_x$, appending the subscript 0 to the values for $r = r_0$ and $x \to -\infty$. Since the gap between the cylinders is small, we shall assume that the corrections $e_x$, $e_y$, and $h$ are small compared with $E_0$ and $H_0$. The intensity $E_0$ is directed along $r$, since on the cylinders $E_x = 0$. As $x \to -\infty$, $E_r = C_2/r \approx C_2/r_0 - C_2y/r_0^2$, and therefore $E_0 = C_2/r_0$, $e_y = -E_0y/r_0$. Similarly, $B_0 = C_1/r_0$, $h = -H_0y/r_0$.

The system (2) can be rewritten in the form

$$\frac{\partial h}{\partial x} = -\frac{\nu}{c} \frac{\partial e_y}{\partial x}, \quad \frac{\partial h}{\partial y} = \frac{\nu}{c} \frac{\partial e_x}{\partial y}, \quad \frac{\partial e_x}{\partial x} - \frac{\partial e_y}{\partial y} = -B'/c \frac{\partial h}{\partial x}$$

Eliminating $\partial e_y/\partial x$ from the system (3), we obtain the system of the two equations

$$\frac{\partial h}{\partial y} - \frac{\nu}{c} \frac{\partial e_x}{\partial x} = -\frac{H_0}{r_0}, \quad \left(1 - \frac{B'}{c^2} \frac{v^2}{c^2}\right) \frac{\partial h}{\partial x} + \frac{\nu}{c} \frac{\partial e_x}{\partial y} = 0$$

We shall describe the discontinuity front by the equation $x = f(y)$ and assume that it differs little from the direction of the $y$ axis, so that $f' \ll 1$ and for the components $n_x$ and $n_y$ of the normal vector we can take the expressions $n_x = 1$ and $n_y = f'(y)$. It