NONEQUILIBRIUM HYPersonic FLOW OF GAS PAST A WING
OF SMALL ASPECT RATIO

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It is well known [1] that nonequilibrium physicochemical processes taking place in gases at high temperature influence the gas-dynamic parameters and aerodynamic characteristics of bodies in hypersonic flight. In the present paper, the thin shock layer method [2-4] is used to consider the problem of nonequilibrium hypersonic flow of gas past a wing of small aspect ratio at an angle of attack. It is shown that the flow component of the vorticity is conserved along the streamlines, and this property is exploited to obtain an analytic solution of the equations of the three-dimensional nonequilibrium shock layer. The influence of the disequilibrium on the thickness of the shock layer and the pressure distribution is investigated.

1. In the coordinate system $x = cx$, $y = cc \tan cy$, $z = cc^{1/2} \tan cz$ attached to the wing, we have in accordance with [1-5] the following equations of the first approximation of the thin shock layer method and boundary conditions on the shock wave and the wing:

$$
Dv = \frac{p_x}{\rho}, \quad Dw = 0, \quad \frac{D\rho}{\rho} + w_x + w_z = 0, \quad \rho = \frac{\mu(1+m)}{mT}, \quad \mu = \left( \sum c_i \right)^n
$$

(1.1)

$$
Dh = 0, \quad h = H(mT_w T, q_1, \ldots, q_N), \quad Dq_n = Q_n (p_n V_n \sin^2 \alpha, mT_w T, q_1, \ldots, q_N), \quad n = 1, 2, \ldots, N
$$

(1.2)

$$
v_x = -S_x + S_w, \quad w_x = -S_w, \quad \rho_x = p_x = -1 - S_x^2 + 2S_w, \quad T_x = 1 + m^{-1}, \quad \mu_x = 1
$$

$$
h_x = 1 + m^{-1} - mh, \quad q_n = q_n = (n = 1, 2, \ldots, N) \quad (y = S(x, z))$$

$$
v_y = B_x + w_x B_z, \quad (y = B(x, z)), \quad v^o = V \sin \alpha, \quad w^o = V \cos \alpha \sin \omega$$

(1.3)

Here, $v^o$ and $w^o$ are the projections of the velocity $V$ onto the axes $y$ and $z$, $p^o$ is the pressure, $\rho^o$ is the density, $T^o$ is the temperature, $h^o$ is the enthalpy, $\mu$ is the molecular weight, $q_1, \ldots, q_N$ is the set of parameters that characterize the composition and state of the gas, $c_i$ are the mass concentrations of the components, $\alpha$ is the specific-heat ratio, $\alpha$ is the angle of attack, $h_f$ is the bound energy of the physicochemical transformations, and $H, Q_1, \ldots, Q_N$ are known functions [5]. The parameter $m \gg 1$, so that the temperature behind the shock reaches the characteristic temperature of the nonequilibrium processes. In contrast to the case of a perfect gas, the system of equations for nonequilibrium flow contains the varying temperature and gas density in the principal term. In accordance with (1.1), an important general property of nonequilibrium flows in a shock layer is the conservation along the streamlines of the flow component of the vorticity, $\omega_x = \epsilon^2 \cos \omega w_x$ divided by the gas density:

$$
D(\omega_x / \rho) = 0
$$

(1.4)

2. Using this property, we obtain analytic expressions for the functions $v, p, M, \omega$ in the system (1.1). Replacing the continuity equation by Eq. (1.4), going over

to the new independent variables \( x, w, z \), and integrating with allowance for (1.2) and (1.3), we obtain

\[
y = B(x, z) + \int_{w'}^w \rho^{-1} \Gamma(w', z-w'x) \, dw'
\]

(2.1)

\[
v = \int_{w'}^w \left[ (w-w') \rho^{-1} \Gamma(w') \rho^{-1} T D^2 p \right] \, dw' + D^2 B - \rho_0^{-1} \Gamma_0 D^2 w_0
\]

(2.2)

\[
p = -1 - S_z^2 + 2 S_z k \int \frac{\rho^{-1} \Gamma(w, z-wx) \, dw'}{\sqrt{\theta}}
\]

(2.3)

Here, \( \Gamma \) is the function that characterizes the vorticity distribution; on the surface of the wing \( (w_b)_x + w_b(w_b)_z = 0 \), or \( \Gamma_b = 0 \).

The function \( S(x, z) \), which describes the shape of the shock wave, and the function \( \Gamma \) are determined from a system of equations with two independent variables:

\[
S(x, z) = B(x, z) + \int_{w'}^w \rho^{-1} \Gamma(w, z-wx) \, dw', \quad \Gamma(-S_z, z+S_wx) = (S_z^2 - S_z)^{-1}
\]

(2.4)

Equations (2.1)-(2.4) contains the function \( \rho \), which must be determined with allowance for the disequilibrium (in a perfect gas, \( \rho = 1 \)).

Since the streamlines are nearly straight, \( x = \text{const} \), for flow past a slender wing of small aspect ratio, we shall determine approximately the temperature, density, and the parameters \( q_n \) by solving the one-dimensional problem of nonequilibrium flow at constant pressure and constant enthalpy. Using the results of [6], we obtain the density distribution of the air in the form

\[
\rho = \begin{cases} 
1 + K \ln \frac{x-x_\infty}{\Delta s} & (x-x_\infty) > \Delta s \\
1 & (0 \leq x-x_\infty) \leq \Delta s 
\end{cases}
\]

(2.5)

where \( K, \Delta s \ll 1 \) are approximation constants [6], and \( x_\infty \) is the abscissa of the point of entry of the streamline into the shock layer, determined by the equation \( w = -S_z(x_\infty, w_\infty) \). We have \( K \ll 1 \), since usually the relative variations of the density associated with the disequilibrium are not large [1]. Analysis of the computational data given in [6] shows, for example, that for \( p_s = 0.33 \times 10^5 \) Pa, \( 5 \leq h_s \leq 30 \) (\( h_s \) is the enthalpy behind the shock in mJ/kg) \( K = 0.115 + (h_s - 10) \times 0.0048 \).

In the variables \( x, x_\infty, z \), the system (2.4) takes with allowance for (2.5) the form

\[
S(x, z) = B(x, z) + \int_{w'}^w \frac{\rho^{-1} \Gamma(w, z-wx) \, dw'}{\sqrt{\theta}}
\]

(2.6)

(2.7)

3. We shall seek a class of particular exact solutions of the system (2.6)-(2.7) in the form

\[
B(x, z) = -\frac{b(x)z^2}{2}, \quad S(x, z) = G(x) + B(x, z), \quad G_b = 0
\]

(3.1)

Under the condition (3.1), Eqs. (2.6) and (2.7) can be solved independently, giving

\[
G(x) = \int F(x, \xi) d_\xi, \quad G(x) = \frac{x^{\frac{b}{1-b}}}{\xi} \ln \frac{x}{\Delta s} \ln \frac{\xi}{b} + \ln \frac{H(x)}{b} + L_2(b) - L_3(H)
\]

(3.2)

In the case \( b(x) = b/x \), which corresponds to flow past a conical wing, we obtain

\[
G(x) = e^{\frac{b}{1-b}} \ln \frac{H(x)}{b} + L_2(b) - L_3(H)
\]

(3.3)