


APPROXIMATE METHOD FOR SOLVING THE KINETIC EQUATION FOR MODERATELY DENSE GASES NEAR A BOUNDARY. SLIP OF A BINARY MIXTURE

M. N. Gaidukov, V. N. Kompaneets, and Yu. I. Yalamov

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The problem of the slip of a binary gas mixture of moderate density along a flat surface is considered. A solution is obtained by a modification of Maxwell's method by means of the Enskog-Thorne kinetic equations for dense gases. The velocities of isothermal, thermal, and diffusion slip for rarefied and moderately dense binary mixtures are compared.

In earlier work, the present authors have obtained an expression for the slip velocity of a single-component gas of moderate density along a flat surface by means of a modification of Maxwell's method. The obtained result differed slightly from the result of [1, 2], in which the problem was solved by the method of half-space moments using the BGK model as collision operator.

The use of models for the collision integrals in the case of rarefied gas mixtures leads to results differing strongly from the experimental data [3, 4], since in this case no allowance is made for various phenomena, for example, thermodiffusion, viscosity and diffusion are inadequately described, etc. Obviously, this is also true for mixtures of moderate density, and therefore in the present paper an attempt is made to obtain an expression for the slip velocity of a binary mixture by a modification of Maxwell's method, which makes it possible to solve the problem without replacing the collision operator by some model.

We consider a binary gas mixture above the plane x = 0. The number densities of the molecules of the components of the mixture, n_i, and the temperature T are distributed nonuniformly along the surface and are described by the expressions

\[ n_i = n_i^0 + \frac{dn_i}{dy} \cdot y \quad (i=1,2), \quad T = T_0 + \frac{dT}{dy} \cdot y \]  \hspace{1cm} (1)

In addition, at large distances from the surface the gas moves with the mass-average velocity

\[ u_m = u_0 + (du/dx)_m \cdot x \]  \hspace{1cm} (2)

where \( u_0 \) is the slip velocity of the gas along the surface.

The gradients \( \frac{dn_i}{dy}, \frac{dT}{dy}, (\frac{du}{dx})_x \) are given, fairly small quantities.

We expand the collision operator of the Enskog–Thorne equation in powers of the small parameter \( \sigma/l \), where \( \sigma \) is the order of the effective diameters of the molecules of the mixture, and \( L \) is the thickness of the Knudsen layer at the surface, this being equal in order of magnitude to the mean free path \( l \) of the molecules. Retaining terms of first order in \( \sigma/l \), we write the stationary Enskog–Thorne equation in the absence of external forces in the form [5]

\[
(v_i \nabla) f_i = \frac{1}{2} \sum_{j=1}^s \chi_0 \int (f_{i'} f_{i''} - f_i f_i) \sigma_0^2(g_{0i} k) d\mathbf{k} d\mathbf{v}_i + \left( \frac{1}{2} \sum_{j=1}^s \chi_0 \int k (v_i \nabla f_i + f_i \nabla v_i) \sigma_0^2(g_{0i} k) d\mathbf{k} d\mathbf{v}_i \right) \frac{1}{2} \sum_{j=1}^s \int (k \nabla \chi_{ij}) (f_{i'} f_{i''} + f_i f_i) \sigma_0^2(g_{0i} k) d\mathbf{k} d\mathbf{v}_i \tag{3}
\]

Here, \( f_i = f_i(r_i, v_i) \) is the distribution function for the molecules of species \( i \); \( f_{i'} = f_{i'}(r_i, v_i') \); \( v_i, v_i' \) are the velocities of molecules of species \( i \) before and after collision, respectively; \( \sigma_0^2 \) is the effective diameter of the molecules of species \( i \); \( g_0 = v_i - v_i' \) is the relative velocity of the molecules of species \( i \) and \( j \); \( k = (r_i - r_j)/|r_i - r_j| \) is the unit vector directed along the line of apsides from molecule \( j \) to molecule \( i \); \( \chi_{ij} \) is the generalized Enskog function that takes into account the probability of collisions of molecules with increasing density of the gas; and \( \beta \) is the distance along the line of apsides from the center of molecule \( i \) to the point at which the function \( \chi_{ij} \) is calculated.

We linearize the problem, i.e., we represent the distribution function as \( f_i(x, y; v_i) = f_i^0(y) [1 + \psi_i(x, v_i)] \), where \( f_i^0 \) is a local Maxwell distribution function. Then the first-order correction to the Maxwell distribution function is the solution of the following system of integrodifferential equations:

\[
v_i \psi_i \frac{\partial f_i^0}{\partial x} + f_i^0 \left[ K_{i}^{(1)} \left( C_i - \frac{5}{2} \right) \frac{d\ln T}{dy} \right] + (-1)^{n_i} \frac{n}{n_i} d_n v_i =
\]

\[
- \sum_{j=1}^s n_j n_i \chi_{0i}(\psi_i + \psi_j) + \sum_{j=1}^s \sigma_0^2 \chi_{0i} \left( \int k f_{i'} f_{i''} (\psi_i + \psi_j) \right) \times
\]

\[
(g_{0i} k) d\mathbf{k} d\mathbf{v}_i + \frac{d\ln T}{dy} \sum_{j=1}^s \sigma_0^2 \chi_{0i} \left( \int k f_{i'} f_{i''} (C_i + C_j) (g_{0i} k) d\mathbf{k} d\mathbf{v}_i \right)
\]

\[
+ \sum_{j=1}^s 2 \left( \chi_{0i} \frac{dn_i}{ny} - \frac{3}{2} \chi_{0i} \frac{d\ln T}{dy} + \frac{\beta}{\sigma_0} \frac{\partial \chi_{0i}}{\partial y} \right) \sigma_0^2 \left( \int k f_{i'} f_{i''} (g_{0i} k) d\mathbf{k} d\mathbf{v}_i \right)
\tag{4}
\]

Here, \( n_i n_i \chi_0(\psi_i + \psi_j) = \int f_{i'} f_{i''} (\psi_i + \psi_j - \psi_{i'} - \psi_{j'}) \sigma_0^2(g_{0i} k) d\mathbf{k} d\mathbf{v}_i \), \( n = n_1 + n_2 \), \( C_i = (m_i/2kT)^{\frac{1}{2}}(v_i - u) \) is the dimensionless proper velocity of the molecules of species \( i \), \( k \) is Boltzmann's constant, and

\[
K_{i}^{(1)} = 1 + \sum_{j=1}^s \frac{12}{5} \frac{b_{ij} \chi_{0i} M_{ij}}{M_i} \], \quad b_0 = \frac{2}{3} \pi n_0 \sigma_0^3, \quad M_0 = \frac{m_i}{m_1 + m_i}
\]

\[
\frac{dn_i}{dy} = \frac{1}{n_i} \left( 1 + b_{ij} \chi_{0i} + 2 b_{ij} \chi_{0i} M_{ij} \right) \frac{d\ln T}{dy} + \frac{1}{n_i} \left( (1 + 2 b_{ij} \chi_{0i}) \frac{dn_i}{dy} + 2 b_{ij} \chi_{0i} \frac{dn_2}{dy} + n_1 \left( b_{ij} \chi_{0i} + \frac{2}{3} \pi n_0 \chi_{0i} \frac{dn_2}{dy} \right) \right)
\tag{5}
\]

The expression (5) for the thermophoretic force, like the collision operator depends on the choice of the point at which the function \( \chi_{ij} \) is calculated. Thus, if \( \chi_{ij} \) is determined at the middle of the segment joining the centers of the colliding molecules, then \( \beta = \sigma_4/2 \) and the expression (6) is equal to \( dy \) of [6]. There are other opinions about the choice of the point for calculating \( \chi_{ij} \) [7, 8]. For example, the function \( \chi_{ij} \) can be calculated at the point at which the molecules touch (then \( \beta = \sigma_4/2 \)) or at the point that is the center of mass of the two colliding molecules (\( \beta = M_i \chi_{0i} \)).

The requirement that \( \sigma/l \) be small imposes a restriction on the gas density: \( b_{ij} \ll 1 \) (i, j = 1, 2), since \( \sigma/l \sim b_{ij} \). This has the consequence that \( (b_0)^n \sim 0 \) (n=2, 3,...), \( b_{ij} \chi_{0i} \sim b_0 \).