Limit theorems are proved by using certain pseudometrics as distances between distributions in the scheme of summation of series of independent random variables with values in a separable Hilbert space.

Introduction

Convergence in mean of the distribution functions of standardized sums of independent random variables (the scheme of growing sums) to the normal distribution function was first considered by Agnew [1]. A survey of the results up to 1970 about global theorems can be found in [2].

In [3] the author has proved global limit theorems containing all known results in this direction. These theorems contain a statement about the convergence of moments of a certain fixed order of sums of independent random variables to the moment of the same order of the limit distribution.

In the present article we give general global limit theorems. In this connection, as a measure of proximity of two distribution functions, we take a certain pseudometric \( \nu(F,G) \). It differs from a metric only in that the triangle inequality is replaced by the following inequality:

\[
\nu(F,G) \leq A(\nu(F,R)+\nu(R,G)),
\]

where \( A > 1 \) is a fixed number. Let us observe that the often-used metric

\[
\sqrt[p]{\int_{-\infty}^{\infty} |F(x)-G(x)|^p \, dx}, \quad p > 1, \quad q > 0,
\]

is a particular case of the pseudometric \( \nu(F,G) \) considered by us. For the definition of this pseudometric, it is necessary to introduce special classes of functions.

**Definition 0.1.** A continuous function \( q(x), x \in (-\infty, \infty) \) belongs to the class \( H \) if it satisfies the following conditions:

1) for each \( h > 0 \)

\[
\inf_{x > h} \frac{q(x)}{x} > 0,
\]
2) for each $\varepsilon > 0$ there exists a number $\delta(\varepsilon) > 0$ such that
\[
\phi(\varepsilon \cdot x) \leq \phi(\varepsilon) \phi(x), \quad x \in (-\infty, +\infty).
\]

The following functions belong to the class $\mathcal{A}$:
\[
\psi_1(x) = |x|^\rho, \quad \rho > 0; \quad \psi_2(x) = (\ln(c + |x|))^\rho, \quad \rho > 0.
\]

**Remark 0.1.** For each function $\Phi \in \mathcal{A}$ there exist two numbers $A(\Phi) = A > 0$ and $B(\Phi) = B > 0$ such that
\[
\Phi(x) \leq A |x|^B
\]
for $|x| \geq 2$.

**Definition 0.2.** A continuous function $\Phi(x), x \in (-\infty, \infty)$, belongs to the class $\mathcal{B}_0$ if it satisfies the following conditions:

1) $\Phi$ is a strictly increasing even function on $[0, \infty)$ such that $\Phi(0) = 0$,
2) $\Phi$ is a convex (downwards) function on $[0, \infty)$,
3) for arbitrary $x$ and $y$ and a certain number $A(\Phi) = A > 1$
\[
\Phi(x + y) \leq A [\Phi(x) + \Phi(y)].
\]
The function $\Phi(x) = |x|^\rho, \rho > 1$ is an example of a function of this class.

**Definition 0.3.** A continuous function $\Phi(x), x \in (-\infty, \infty)$, belongs to the class $\mathcal{B}_2$ if it satisfies the following conditions:

1) $\Phi$ is a strictly increasing even function on $[0, \infty)$ such that $\Phi(0) = 0$,
2) $\Phi(x + y) \leq \Phi(x) + \Phi(y)$ for arbitrary $x$ and $y$,
3) For arbitrary $x$ and arbitrary $0 < c \leq 1$ and a certain fixed number $A(\Phi) = A > 0$
\[
\Phi(cx) \leq c^d \Phi(x).
\]
The function $\Phi(x) = |x|^\rho, \rho < d \leq 1$ is an example of a function of this class.

**Remark 0.2.** For each function $\Phi \in \mathcal{B}_0 \cup \mathcal{B}_2$ there exist two positive numbers $A > 0$ and $B > 0$ such that
\[
\Phi(x) \leq B |x|^A.
\]
for $|x| \geq 2$.

We define a pseudometric on the set of distribution functions by the rule
\[
\nu(F, G) = \int_{-\infty}^{\infty} \Phi(F(x) - G(x)) q(x) \, dx,
\]
where $\Phi \in \mathcal{A}$ and $Q \in \mathcal{B}_0 \cup \mathcal{B}_2$ are fixed functions.

Let $H$ be a separable real Hilbert space. As usual, $\|x\|, (x, y)$, and $\theta$ will denote the norm of an element $x \in H$, the scalar product of elements $x, y \in H$, and the zero element of $H$ respectively.

By a random element we will mean a measurable mapping of a probability space $\{\Omega, \mathcal{A}, \rho\}$ into a measurable Hilbert space $\{H, \mathcal{B}\}$, where $\mathcal{B}$ is the $\sigma$-algebra of Borel sets in $H$. 

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