PROPAGATION OF SPHERICAL SHOCK WAVES
IN AN EXPONENTIAL MEDIUM WITH
RADIATION HEAT FLUX

J. B. SINGH and S. K. SRIVASTAVA
Dept of Mathematics and Physics, S.G.R. Postgraduate College, Dobhi, Jaunpur, U.P., India

(Received 13 April, 1982)

Abstract. In this paper propagation of spherical shock waves with radiation heat flux is considered in an exponentially increasing medium. The shock wave moves with variable velocity and the total energy of the wave is variable. For different values of radiation parameter, the numerical solution has been made and the nature of the field variables are illustrated by the tables.

1. Introduction

Grover and Hardy (1966), Hayes (1968), Ray and Bhowmick (1974), and Verma and Singh (1979 and 1980) have discussed the propagation of shock waves in a medium where density varies exponentially. These authors have not taken radiation effects into account. Laumbach and Probstein (1970) has only studied the radiation effects on shock wave in an exponential medium.

In the present paper a solution, for the propagation of spherical shock waves with radiation heat flux in an exponentially increasing medium under uniform pressure, has been obtained. The disturbance has been assumed to be generated on account of a point explosion in a ideal gas. It is assumed that the gas to be grey and opaque and the shock to be transparent and isothermal. Radiation pressure and energy have been neglected. The total energy of the wave is nonconstant. The gas ahead of the shock is assumed to be at rest. Effects of viscosity, magnetic field and gravitation have not been taken into account. Following Singh and Srivastava (1981), this problem may be extended to the case of spheroidal symmetry or axial symmetry in an exponential medium. The integrals to the equations governing the flow behind the shock form a set of non-similarity solutions.

2. Equations of Motion and Boundary Conditions

The flow behind a spherical shock surface taking radiation flux into account, is governed by equations

$$\frac{\partial p}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (p ur^2) = 0,$$

(1)

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0,$$

(2)

Copyright © 1982 by D. Reidel Publishing Co., Dordrecht, Holland, and Boston, U.S.A.
\[
\frac{1}{(\gamma - 1)} \left[ \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} \right] - \frac{\gamma p}{(\gamma - 1) p} \left[ \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} \right] + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 F) = 0 ,
\]

where:

- \( u \) = particle velocity in the radial direction,
- \( \rho \) = density in the disturbed fluid at a distance \( r \),
- \( p \) = pressure at a distance \( r \),
- \( F \) = radiative heat flux,
- \( \gamma \) = ratio of specific heats and is taken constant.

Assuming local thermodynamic equilibrium, and taking Rosseland's diffusion approximation, we have

\[
F = -\frac{c \mu}{3} \frac{\partial}{\partial r} \left( \alpha T^4 \right) ,
\]

where \( \alpha c/4 \) is the Stefan–Boltzmann constant; \( c \), the velocity of light; and \( \mu \), the mean-free path of radiation, is a function of density \( \rho \) and absolute temperature \( T \). Following Wang (1966), we take

\[
\mu = \mu_0 \rho^{-\alpha} T^\beta ,
\]

where \( \alpha \) and \( \beta \) are constants. As the gas is taken ideal, the equation of state is given by

\[
p = \Gamma \rho T ,
\]

where \( \Gamma \) is the gas constant.

The disturbance is headed by an isothermal shock surface and, hence, the conditions across it are

\[
\rho_1 (V - u_1) = \rho_0 V = m_1 ,
\]

\[
p_1 - p_0 = m_1 u_1 ,
\]

\[
\frac{\gamma p_1}{(\gamma - 1) \rho_1} + \frac{1}{2} (V - u_1)^2 - \frac{F_1}{m_1} = \frac{\gamma p_0}{(\gamma - 1) \rho_0} + \frac{1}{2} V^2 ,
\]

\[
T_1 = T_0 ;
\]

where \( \alpha c/4 \) is the Stefan–Boltzmann constant, \( C \), the velocity of light; and \( \mu \), the shock front, respectively, and \( V (= dR/dt) \) denotes the velocity of shock surface at \( r = R(t) \). From Equations (7)-(10) we get

\[
u_1 = \left[ 1 - \frac{1}{\gamma M^2} \right] V ,
\]

\[
\rho_1 = \gamma M^2 \rho_0 ,
\]

\[
p_1 = \rho_0 V^2 ,
\]

\[
F_1 = \frac{1}{2} \left[ \frac{1}{\gamma M^2} - 1 \right] \rho_0 V^3 ,
\]