For any sequence \( \{b_n\} \) such that \( \sum_{n=1}^{\infty} b_n^2 = \infty \), a uniformly bounded system \( \{\Phi_n(x)\} \), orthonormal on \([0, 1]\), is constructed such that the series \( \sum_{n=1}^{\infty} b_n \Phi_n(x) \) diverges to \( +\infty \) on some set \( E \subset [0, 1] \), \( 0 < \text{mes} \ E < 1 \), for any order of the terms.

1. Introduction. Let \( A \) denote the class of series
\[
\sum_{n=1}^{\infty} a_n \phi_n(x),
\]
where \( \{\phi_n(x)\} \) is any system of functions, orthonormal and uniformly bounded on \([0, 1]\), and \( \{a_n\} \) is any sequence of real numbers.

P. L. U'lyanov [1] gave an example of a series (1) of the class \( A \) which diverges to \( +\infty \) on a set \( E \subset [0, 1] \), \( 0 < \text{mes} \ E < 1 \), for any order of the terms. He also posed the following question: Is there a series (1) of the class \( A \) which diverges to \( +\infty \) almost everywhere or on a set of positive measure whose coefficients tend to zero, and what is the maximum rate at which the coefficients can tend to zero?

This problem was partially solved by the authors in [3] for the case of divergence almost everywhere, and the coefficients of the series constructed satisfy the condition
\[
a_n = O \left( \frac{\ln^{2} n}{\sqrt{n}} \right).
\]
A. M. Olevskii [4] described a series (1) of the class \( A \) converging to \( +\infty \) almost everywhere in which
\[
a_n = O \left( \frac{1}{\sqrt{n}} \right).
\]

In the present note we give a complete answer to the above problem for the case of divergence to \( +\infty \) on a set \( E \subset [0, 1] \), \( 0 < \text{mes} \ E < 1 \) in the following theorem.

**Theorem.** Let the sequence of numbers \( \{b_n\} \) be such that \( \sum_{n=1}^{\infty} b_n^2 = \infty \). There is a system of functions \( \{\Phi_n(x)\} \) \( (|\Phi_n(x)| \leq M) \) defined on \([0, 1]\) such that the series \( \sum_{n=1}^{\infty} b_n \Phi_n(x) \) diverges to \( +\infty \) on a set \( E \subset [0, 1] \), \( 0 < \text{mes} \ E < 1 \), for any order of the terms.

**2. D. E. Men'shov's Lemmas Concerning Orthogonalization in a Longer Interval.**

**Lemma 1.** Let \( a_{\sigma} \) be real numbers defined for subscripts \( s \) and \( \sigma \), such that \( 1 \leq s \leq r \), and \( 1 \leq \sigma \leq r \), where \( |s-\sigma| = p \), \( 1 \leq p \leq r-1 \), \( a_{\sigma} = a_{\rho} \), and \( p \) is a fixed positive integer. Let \( \beta_p \) be the largest of the numbers \( |a_{\sigma}| \) and let \([c, d]\) be an arbitrary segment with \( d-c > 2\beta_p \) and \( c = x_1 < \ldots < x_n = d \) any finite decomposition of this segment.

Then a function \( \Phi_{1}(p)(x), 1 \leq i \leq r \), can be defined on \([c, d]\) possessing the following properties:
1) the functions \( \Phi_{1}(p)(x), 1 \leq i \leq r \), are piecewise constant on \([c, d]\);
2) \( \int_c^d \Phi_1^{(p)} \Phi_2^{(p)} \, dx = -a_{s}, \, |s - \sigma| = p, \, 1 \leq s; \, \sigma \leq r; \)

3) \( |\Phi_i^{(p)}(x)| = 1, \, x \in [c, d], \, 1 \leq i \leq r; \)

4) \( \int_{c}^{d} \Phi_1^{(p)} \Phi_2^{(p)} \, dx = 0 \) for \( s \neq \sigma, \, |s - \sigma| = p, \, 1 \leq s, \, \sigma \leq r; \)

5) \( \int_{a_k}^{x_{k+1}} \Phi_1^{(p)} \, dx = 0, \, 1 \leq s \leq r, \, 1 \leq k \leq n - 1. \)

This result is based on the following.

**Lemma 2.** Let \( f_i(x), \, 1 \leq i \leq n, \) be piecewise-constant functions defined on \([a, b]\) and let \( a = x_1 < x_2 < \ldots < x_n = b \) be any finite decomposition of \([a, b]\). Then for any \( \alpha, \) where \( |\alpha| < b - a, \) there are two functions \( \psi_1(x) \) and \( \psi_2(x), \) on \([a, b]\) satisfying the following conditions:

1) \( \psi_1(x) \) and \( \psi_2(x) \) are piecewise constant on \([a, b]\),

2) \( |\psi_1(x)| = |\psi_2(x)| = 1, \, x \in [a, b]; \)

3) \( \int_{a}^{b} \psi_1 \psi_2 \, dx = -x; \)

4) \( \int_{a}^{b} \psi_{i} f_k \, dx = 0, \, i = 1, 2, \, 1 \leq k \leq n; \)

5) \( \int_{a}^{b} \psi_{i} \, dx = 0, \, 1 \leq i \leq n - 1, \, k = 1, 2. \)

D. E. Men'shov [5] proved a lemma (Lemma 1, p. 104) differing from Lemma 1 only in that its formulation does not include condition 5), and it is assumed that the numbers \( a_{\sigma \sigma} \) are nonzero. The proof of this lemma is based on an assertion differing from Lemma 2 only in that condition 5) is absent.

Lemma 1 can be proved by the reasoning used to prove Lemma 1 of [5] and by using Lemma 2 instead of the assertion made in the same work and referred to above. We thus confine ourselves here to giving a proof of Lemma 2.

Let \( a = y_1 < y_2 < \ldots < y_m = b \) be a decomposition of \([a, b]\) such that the intervals \((y_i, y_{i+1}), \, i = 1, 2, \ldots, m-1,\) are intervals of constancy of all the functions \( f_k(x), \, k = 1, \ldots, n; \) further let \( y_{i_0} = a + |\alpha| \) for some \( i_0 \) and let segments of the form \([y_k, y_{k+1}]\) be sums of segments of the form \([y_i, y_{i+1}]. \) We divide up the interval \((y_i, y_{i+1})\) into four equal parts by the points \( y_i, y_i, y_i, y_{i+1} \) \( (y_i < y_i < y_i < y_{i+1}) \) and define our two functions as follows:

\[
\psi_1(x) = \begin{cases} 
1, & x \in [y_i, y_i], \, 1 \leq i \leq m, \\
-1, & x \in [y_i, y_i], \, 1 \leq i \leq m, \\
-\psi_1(x) \, \text{sign} \, x, & a \leq x \leq y_i, \\
1, & x \in [y_i, y_i], \, y_i < x \leq y_i, \\
-1, & x \in [y_i, y_i], \, y_i < x \leq y_i, \\
-\psi_1(x) \, \text{sign} \, x, & a \leq x \leq y_i, \\
1, & x \in [y_i, y_i], \, y_i < x \leq y_i, \\
-1, & x \in [y_i, y_i], \, y_i < x \leq y_i.
\end{cases}
\]

\[
\psi_2(x) = \begin{cases} 
1, & x \in [y_i, y_i], \, 1 \leq i \leq m, \\
1, & x \in [y_i, y_i], \, y_i < x \leq y_i, \\
-1, & x \in [y_i, y_i], \, y_i < x \leq y_i.
\end{cases}
\]

It is easily checked that \( \psi_1(x) \) and \( \psi_2(x) \) satisfy all the conditions of Lemma 2.

**Lemma 3.** Let the functions \( f_i(x), \, 1 \leq i \leq r, \) be square-integrable on \([a, b]\) and let \( \|a_{\sigma \sigma}\| \) be the matrix of scalar products of these functions:

\[
a_{\sigma \sigma} = \sum_{i=1}^{r} f_i \, f_i \, dx, \, 1 \leq s, \, \sigma \leq r. \quad (4)
\]

Let

\[
\beta_p = \max \{|a_{\sigma \sigma}|, |\sigma - s| = p\}, \, 1 \leq p \leq r - 1. \quad (5)
\]

Then if \( e - b > \sum_{p=1}^{r-1} 2^{\beta_p} \) and \( b = x_1 < x_2 < \ldots < x_n = c \) is any finite decomposition of \([b, c]\), the functions \( f_i(x), \, 1 \leq i \leq r, \) can be defined on the interval \((b, c]\) so that the following conditions hold: