The following proposition is proved below: Hochschild cohomologies of maximal order of a simple central algebra over the field of algebraic numbers are periodic with period 2.

**Introduction.** It was shown in [1] that the ring of integral elements of the field of algebraic numbers has the property of periodic cohomology. The period is equal to two. Below, this result is extended to the case of maximal orders of simple central algebras.

1. **Auxiliary Results.** Let \( \mathfrak{A} \) be a semisimple algebra over the field of rational numbers \( \mathbb{Q} \), \( \mathbb{Z} \) the ring of integral rational numbers, and \( \Lambda \) a \( \mathbb{Z} \)-ring which is a Frobenius \( \mathbb{Z} \)-algebra (see [1]). For a \( \Lambda \)-bi-module \( A = 1 \cdot A \cdot 1 \) we shall denote by \( H_n(\Lambda, A) \) (\( H^n(\Lambda, A) \)) the Hochschild homology (cohomology) groups of the ring \( \Lambda \) with coefficients in \( A \) (see [2]). On the modules \( \text{Hom}_\mathbb{Z}(\Lambda, A) \) and \( A \otimes \mathbb{Z} A \) it is possible to define operators belonging to the ring \( \Lambda \):

\[
(\lambda_1b_2)(\mu) = \lambda_1(\lambda_2\mu), \\
\lambda_1(\lambda_2a)\mu = \lambda_2a \otimes \mu b_2, \\
f \in \text{Hom}_\mathbb{Z}(\Lambda, A), \lambda_1, \lambda_2, \mu \in \Lambda, a \in A.
\]

The obtained \( \Lambda \)-bimodules are isomorphic and homologically trivial. Let us consider the exact sequences

\[
0 \rightarrow A \rightarrow \text{Hom}_\mathbb{Z}(\Lambda, A) \rightarrow J(\Lambda) \rightarrow 0, \\
(\text{mod}) \quad (\lambda) = a \Lambda, \quad J(\Lambda) = \text{Coker}\, \theta, \\
0 \rightarrow I(\Lambda) \rightarrow A \otimes \mathbb{Z} \Lambda \rightarrow A \rightarrow 0, \\
\zeta(\lambda \otimes a) = a \Lambda, \quad I(\Lambda) = \text{Ker}\, \zeta.
\]

From the exact homologic sequences for the triples (1) and (2) we obtain the isomorphisms

\[
H^{m-1}(\Lambda, A) \simeq H^m(\Lambda, I(\Lambda)), \\
H_{m-1}(\Lambda, A) \simeq H_m(\Lambda, J(\Lambda)) \\
(\text{mod} \, \geq 2).
\]

Negative cohomology groups can be defined recursively:

\[
H^n(\Lambda, A) = H^1(\Lambda, I(\Lambda)), \\
H^{-m}(\Lambda, A) = H^{-(m-1)}(\Lambda, I(\Lambda)) \quad (\text{mod} \, \geq 1).
\]

As a result we obtain a two-sided connected sequence of covariant functors \( H_m(\Lambda, \cdot) \), where \( m \) is an integer [1].

2. **Tensor Products of Two Normal Isomorphic Fields.** Here we shall present some results concerning the structure of bimodules over the ring of integral elements of an unbranched local field. At first let us consider a more general case. Let \( k \) be a field of characteristic zero, and \( K \) its normal extension with a Galois group \( G \).
PROPOSITION 2.1 (Deuring's theorem). In the \( k\)-algebra \( K \otimes_k K \) there exists a set of orthogonal idempotents
\[ e_\sigma, \sigma \in G, \]
such that
\[ K \otimes_k K \cong \sum_{\sigma \in G} K e_\sigma, \quad x e_\sigma = x^2 e_\sigma, \quad \sigma \in G. \]
Here
\[ K_1 = K \otimes 1, \quad x_1 = x \otimes 1, \quad x_2 = 1 \otimes x, \quad x \in K. \]
The proof can be found in [4].

Let \( \theta \) be a primitive element of the field \( K = k(\theta) \), and \( \varphi(t) \in k[t] \) an irreducible polynomial whose root is \( \theta \). Then the elements \( e_\sigma \) can be calculated by the formula
\[ e_\sigma = \frac{1}{\varphi'(\theta_\sigma^2)} \prod_{\tau \neq \sigma} (\theta_\tau - \theta_\sigma), \quad \sigma \in G. \tag{4} \]

Let us apply the theorem to the case that \( K \) is an unbranched extension of the field of \( p \)-adic numbers \( k = k_p \). Let \( \mathfrak{o} \) and \( \mathfrak{m} \) be rings of integral elements of the fields \( k \) and \( K \). In an unbranched extension it is possible to select the primitive element \( \theta \) in such a way that \( \theta^\sigma = \theta^r \) (mod \( \pi \)) (\( \pi \) is primitive in \( K \)). Hence
\[ \Omega \otimes \Omega = \sum \Omega_\sigma e_\sigma. \]

Therefore,
\[ \Omega \otimes \Omega = \sum \Omega_\sigma e_\sigma. \]

Let us consider an \( \Omega \)-bimodule \( A \). It can be regarded as a left module over an enveloping algebra \( \Omega^\mathfrak{g} \), which coincides (in view of the commutativity of \( \Omega \)) with the tensor product \( \Omega \otimes \Omega = \sum \Omega_\sigma e_\sigma \). Thus we obtain a decomposition of \( A \) into a direct sum of submodules \( e A \): \( A = \sum \Omega_\sigma e_\sigma A \). The elements \( a \in e_\sigma A \) are characterized by the property \( \omega a = a \omega^\sigma, \omega \in \Omega \).

3. Cohomologies of Maximal Order in Division Algebras. Let \( \mathfrak{g} \) be a central division algebra of rank \( n^2 \) over the field of \( p \)-adic numbers \( k \), \( \mathfrak{o} \) a ring of integral elements \( k \), \( A \) the maximal order of the algebra, and \( \Pi \in A \) a primitive element. As is generally known, \( \Pi \mathfrak{g} = \pi \), where \( \pi \) is primitive in \( k \) [3]. By \( K \) we shall denote the maximal unramified extension of \( k \), contained in \( \mathfrak{g} \), and by \( \Omega \) the ring of integral elements of the field \( K \). Let \( A \) be a two-sided \( \Lambda \)-module. Since \( \Omega \subset \Lambda \), \( A \) can be also regarded as an \( \Omega \)-bimodule. Hence there exists a decomposition \( A = \sum \otimes \Omega e_\sigma A \). The Galois group of the extension \( K/k \) is a finite cyclic group of order \( n \). Its generatrix is the automorphism \( x \mapsto x^\Pi = \Pi^{-1} x \Pi, x \in K \). Therefore the decomposition of the module \( A \) can be written in the form
\[ A = \sum \otimes \Omega e_\sigma A \quad \sum \otimes \Omega e_\sigma A \]
where \( A^{(k)} = e_\sigma A \) is an \( \Omega \)-submodule of \( A \) whose elements are specified by the relation \( \omega a = a \omega^{\Pi^k}, \omega \in \Omega \). Such elements are said to be homogeneous of degree \( k \): \( \deg a = k \). Let us introduce operations over the \( \Lambda \)-bimodule \( A \):
\[ a a = \sigma a a = \sum_{i=0}^{n-1} \Pi^{-n-k} a \Pi^k, \quad a \in A. \tag{5} \]
\[ a a = \tau a a = \Pi a - a \Pi, \quad a \in A. \tag{6} \]

Let us note that if \( a \in A \) is homogeneous of degree \( k \), \( 0 \leq k \leq n - 1 \), then
\[ \deg \sigma a = k - 1, \quad \deg \tau a = k + 1 \pmod{n}. \]
It is evident that \( \sigma \tau = \tau \sigma = 0 \).

In Section 1 we already introduced operators over a ring \( \Lambda \) on a module \( \hat{\Lambda} = \text{Hom}_\mathfrak{o}(\Lambda, A) \). They can be defined in yet another way: \( (\lambda_4 f \lambda_2)(\mu) = f(\mu \lambda_2) \lambda_2, f \in \hat{\Lambda}, \lambda_4, \lambda_2, \mu \in \Lambda. \) The thus-obtained \( \Lambda \)-bimodule