Some properties of compact manifolds, on which there are no meromorphic functions except constants, are proved. In particular, the theorem on the finiteness of the number of analytic sets with codim = 1 on such manifolds is proved.

In what follows, \( X \) is a connected compact \( n \)-dimensional complex manifold, which has no meromorphic functions except constants (obviously \( n < 1 \)).

**Proposition 1.** Let \( L \) be a locally trivial vector bundle over \( X \) of rank \( m \). Then the space of meromorphic sections of \( L \) has dimension \( \leq m \).

**Proof.** We assume the contrary, i.e., let \( e_1, \ldots, e_{m+1} \) be linearly independent meromorphic sections of \( L \). For each point \( x \in X \), for which \( e_1(x), \ldots, e_{m+1}(x) \) are defined, the dimension of the linear span of \( (e_1(x), \ldots, e_{m+1}(x)) \) we shall denote by \( r(x) \); let \( r = \max r(x) \). Then \( r \leq m \). There exists a proper analytic set \( M \subseteq X \) and a collection of indices \( i_1, \ldots, i_r \), such that for any point \( x \in X \), \( M \) is defined and \( e_{i_1}(x), \ldots, e_{i_r}(x) \) and \( \dim (e_{i_1}(x), \ldots, e_{i_r}(x)) = r \); we can assume that \( i_1 = 1, \ldots, i_r = r \). Consequently, there exist holomorphic functions \( a_1, \ldots, a_r \), defined in \( X - M \), such that

\[
a_1(x) \cdot e_1(x) + \cdots + a_r(x) \cdot e_r(x) = e_{r+1}(x)
\]

for each point \( x \in X - M \). We shall show now that \( a_1, \ldots, a_r \) extend to holomorphic functions on \( X \). Let \( x \in M \), \( U \) be a neighborhood of the point \( x \), in which \( O(L) = (O(U))^m \). \( e_j = (b_{j1}, \ldots, b_{jm}) \), \( j = 1, \ldots, m+1 \) (\( b_{ij} \) are meromorphic functions). Then in \( U - U \cap M = V \), we have

\[
a_i b_{i1} + \cdots + a_i b_{ir} = b_{i(r+1)}, \quad i = 1, \ldots, r.
\]

One can assume that in \( V - N \), where \( N \) is a proper analytic set in \( V \), the function

\[
\Delta = \begin{vmatrix} b_{11} & b_{12} & \cdots & b_{1r} \\ \vdots & \vdots & \ddots & \vdots \\ b_{r1} & b_{r2} & \cdots & b_{rr} \end{vmatrix}
\]

is nowhere zero (the functions \( b_{ij} \), \( i, j \leq r \) are defined in \( U - U \cap M \)). Then \( a_i = \Delta_i / \Delta \), where

\[
\Delta_i = \begin{vmatrix} b_{i1} & b_{i2} & \cdots & b_{ir} \\ b_{i1} & b_{i2} & \cdots & b_{ir} \\ \vdots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{ir} \end{vmatrix}
\]

is the \( i \)-th row

Thus, \( a_1, \ldots, a_r \) are meromorphic functions on \( X \), so \( a_1, \ldots, a_r \) are constants, and consequently, \( e_1, \ldots, e_{m+1} \) are linearly dependent. The proposition is proved.

PROPOSITION 2. Let \( L \) be a locally trivial vector bundle over \( X \) of rank \( m \); \( e_1, \ldots, e_s \) be linearly independent meromorphic sections of \( L \). Then there exists a proper analytic set \( M \subset X \) such that for each point \( x \in X - M \), \( e_1(x), \ldots, e_s(x) \) are defined and \( \dim(e_1(x), \ldots, e_s(x)) = s \).

Proof. One must argue exactly as in the proof of Proposition 1.

THEOREM (cf. [1], Theorem 5.1). The number of irreducible hypersurfaces in \( X \) is finite and not greater than \( n + h^{1,1} - h^{1,0} \), where \( h^{p,q} = \dim H^q(X, \Omega^p_X) \).

Proof. The vector \( \mathbb{C} \)-space \( \text{Div}(X) \otimes \mathbb{C} \), where \( \text{Div}(X) \) is the group of divisors on \( X \), will be denoted by \( \mathcal{D} \). We denote by \( \mathcal{O}_X^1 \) the sheaf of \( \mathcal{C} \)-moduli consisting of germs of meromorphic functions of the form \( (cdw/w) + \beta \), where \( w \) is a germ of a holomorphic function, \( c \) is a constant, \( \beta \) is a germ of a holomorphic 1-form. We consider the exact sequence of sheaves

\[
0 \rightarrow \mathcal{O}_X^1 \rightarrow \mathcal{O}_X^1 \rightarrow \mathcal{F} \rightarrow 0,
\]

where \( \mathcal{F} \) is the quotient-sheaf. It is easy to see that \( H^q(X, \mathcal{F}) = \mathcal{D} \). On the other hand, \( H^q(X, \mathcal{O}_X^1) \) is a subspace of the space of meromorphic sections of the sheaf \( \mathcal{O}_X^1 \), and so, by virtue of Proposition 1, \( \dim H^q(X, \mathcal{O}_X^1) \leq n \). Hence, \( \mathcal{D} \ll n + h^{1,1} - h^{1,0} \). The theorem is proved.

COROLLARY. If \( X \) is a complex torus, then there are no hypersurfaces in \( X \).

Otherwise, making translations of a single irreducible hypersurface, we would get an infinite number of irreducible hypersurfaces.

PROPOSITION 3. Let \( X \) be Kähler. Then we have:

(i) the Albanese map \( s: X \rightarrow \text{Alb}(X) \) is surjective; \( \text{Alb}(X) \) is a manifold, without meromorphic functions, in particular, \( \dim \text{Alb}(X) \neq 1 \);

(ii) if \( \dim \text{Alb}(X) = n \), then \( s: X \rightarrow \text{Alb}(X) \) is bimeromorphic.

Proof. From Proposition 2, it follows that \( s: X \rightarrow \text{Alb}(X) \) is surjective; \( \text{Alb}(X) \) has no nonconstant meromorphic functions on \( X \). Now let \( \dim \text{Alb}(X) = n \), \( \omega_1, \ldots, \omega_n \) be a basis of homogeneous holomorphic forms on \( X \). Then by virtue of Proposition 2, there exists a proper analytic set \( M \subset X \) such that for any point \( x \in X - M \), \( \omega_1(x) \wedge \ldots \wedge \omega_n(x) \neq 0 \); let \( N \) be equal to the image of \( M \) under the Albanese map. Then the map

\[
s: X \rightarrow s^{-1}(\lambda) \rightarrow \text{Alb}(X) \rightarrow N
\]

is an unbranched covering, and by virtue of the corollary, \( \text{codim } N \geq 2 \). One has the commutative diagram

\[
\begin{array}{ccc}
\pi_1(X - s^{-1}(\lambda)) & \rightarrow & \pi_1(\text{Alb}(X)) \\
\downarrow s_* & & \downarrow s_* \\
\pi_1(\text{Alb}(X) - N) & \rightarrow & \pi_1(\text{Alb}(X))
\end{array}
\]

It is easy to see that

\[
\pi_1(X - s^{-1}(\lambda)) \rightarrow \pi_1(\text{Alb}(X)), s_*: \pi_1(X) \rightarrow \pi_1(\text{Alb}(X))
\]

are epimorphisms; on the other hand, since \( N \geq 2 \), then

\[
\pi_1(\text{Alb}(X) - N) \rightarrow \pi_1(\text{Alb}(X))
\]

is an isomorphism. Consequently,

\[
s_*: \pi_1(X - s^{-1}(N)) \rightarrow \pi_1(\text{Alb}(X) - N)
\]

is an isomorphism, and hence the map

\[
s: X - s^{-1}(N) \rightarrow \text{Alb}(X) - N
\]

is an isomorphism. The proposition is proved.

Remark. In the proof of (ii), we followed Kodaira (cf. [2], proof of Theorem 11).