σ-admissible pairs \(\{\xi_i, \beta_i\}_1^{\alpha}, \{\eta_j, \delta_j\}_1^{\beta}\) and σ-admissible ordered pairs \((\eta_i, \theta_i)_1^{\alpha}, \{\xi_i, \beta_i\}_1^{\beta}\). It is assumed that all \(r + 2s + 2t\) of the subsets occurring here are pairwise disjoint and in addition \(|\xi_i| \geq \Delta, |\beta_j| \geq \delta; |\eta_i^2 | = \beta, |\delta_i| \geq \Delta\). Such a collection of \(r\) subsets and \(s + 1\) pairs of subsets we call a σ-system and we denote it by \(\kappa\). With each σ-system \(\kappa\) there is associated a subgroup

\[G(\kappa) = \langle \Theta(\xi_i), \Gamma(\eta_j, \theta_i) , \Delta(\eta_i, \theta_i) \rangle.\]

It is clear that all \(G(\kappa)\) are D-complete subgroups. If \(\kappa\) is the empty σ-system, then \(G(\kappa) = D\).

**THEOREM.** We consider the set of pairs \((\sigma, \kappa)\), where \(\sigma\) is an arbitrary D-net of order \(n\) and \(\kappa\) is an arbitrary σ-system. The map

\[G(\sigma, \kappa) \rightarrow G(\sigma, \kappa) = G(\sigma) \cdot G(\kappa)\]

defines a bijective correspondence between all pairs \((\sigma, \kappa)\) and all D-complete intermediate subgroups in \(G(\sigma, \kappa)\).

LITERATURE CITED


SUBGROUPS OF THE UNITARY GROUP OVER A DYADIC LOCAL FIELD

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Let \(K/k\) be an unramified quadratic extension of a dyadic local field of characteristic \(\neq 2\). In this paper there is studied the lattice of subgroups of the classical unitary group \(U(n, K)\), containing the subgroup \(D\) of diagonal unitary matrices. The pronormality of the subgroup \(D\) in the group \(U(n, K)\) is proved and a constructive description is obtained of its unique fan. Analogous results are formulated for the "nonclassical" unitary group \(U^*(n, K)\) corresponding to a Hermitian form with matrix \(\text{diag}(1, \ldots, 1, \pi)\) (\(\pi\) is a prime element of the field \(K\)).

1° In the present paper, for the unitary group defined over an unramified quadratic extension of a dyadic local field, there is described the lattice of the subgroups containing the group of diagonal unitary matrices. In [5] the analogous subgroups are described in the unitary group over an unramified quadratic extension of a local nondyadic field. The case of the classical unitary group over the field of complex numbers is considered in [4].

Since the course of the arguments of the present paper coincides to a large extent with the arguments in [5], we shall not give complete proofs of all assertions; nevertheless we shall try to point out the basic points.

Let \(k\) be a discrete normed complete field with finite residue field of characteristic 2 (a dyadic local field). Throughout this paper we shall assume that the characteristic of the
field $k$ is equal to zero; in other words, $k$ is a finite extension of the field $\mathbb{Q}_2$ of $2$-adic numbers. Suppose, further, $K/k$ is an unramified quadratic extension. The image of the element $\xi \in K$ under the nonidentity automorphism of $K/k$ we denote by $\xi$.

We denote by $U = U(n, K)$ the unitary group of matrices of degree $n \geq 2$ over $K$, i.e., the set of those matrices $\alpha$ from $GL(n, K)$, for which $\alpha^* = e$. (Here $\alpha^*$ is the "Hermitian adjoint" matrix of $\alpha$, $(\alpha^*)_{ij} = \alpha_{ji}$; $e$ is the identity matrix of order $n$.) Suppose, further, $D = DU(n, K)$ is the subgroup of diagonal unitary matrices

$$\text{diag}(\xi_1, \ldots, \xi_n), \quad N(\xi_i) = 1 \quad (1 \leq i \leq n),$$

where by $N(\xi) = \xi \xi^*$ we denote the norm of the element $\xi \in K$ with respect to $k$.

Our problem consists of describing the structure of the intermediate subgroups $H$ of the group $U$, i.e., subgroups for which

$$DU(n, K) \leq H \leq U(n, K).$$

The basic result of the present paper is the following. Let $k$ be a dyadic local field, $K$ be an unramified quadratic extension of it and $O$ be the ring of integral elements of the field $K$. To each intermediate subgroup $H$ of the group $U(n, K)$ there corresponds uniquely the symmetric net $\sigma$ of order $n$ of $O$-modules of the field $K$ such that

$$U(\sigma) \leq H \leq N(\sigma),$$

where by $N(\sigma)$ we denote the normalizer of the net subgroup $U(\sigma)$ in the group $U$. (For the definition and properties of nets and net subgroups, cf. [1, 2].)

Let $\rho \to (\rho)$ be the natural imbedding of the symmetric group $S_n$ in $U(n, K)$. By $P(\sigma)$ we denote the image under this imbedding of the subgroup consisting of those permutations $\sigma \in S_n$ for which $\sigma^2 = \sigma$ (cf. [1, Sec. 3]). For the normalizer $N(\sigma)$ one has the decomposition $N(\sigma) = P(\sigma) U(\sigma)$.

In the terms of [3] the subgroup $D$ of diagonal unitary matrices is a fan subgroup in $U(n, K)$. Moreover, it is pronormal in $U(n, K)$. The basis subgroups of the fan (unique due to the pronormality of $D$) are the net subgroups $U(\sigma)$. A principal section of the fan is isomorphic with the symmetric group $S_n$.

We shall use the following notation throughout the entire paper:

- $k$ is a dyadic local field, i.e., a finite extension of the field $\mathbb{Q}_2$ of $2$-adic numbers;
- $K$ is an unramified quadratic extension of the field $k$;
- $\mathcal{O}$ is the ring of integral elements of the field $K$;
- $C$ is the (unique) maximal ideal of the ring $\mathcal{O}$;
- $v$ is the exponent of the field $K$ [$v(\xi) = m$ means that $\xi \in \mathcal{O}^{\mathbb{Q}_2} / \mathcal{O}^{\mathbb{Q}_2}$];
- $\mathbb{F}_q$ is the finite residue field of the local field $k$, containing $q$ ($q = 2^f$) elements.

For short we shall call the dyadic local field $k$ completely ramified, if it is a completely ramified extension of the field $\mathbb{Q}_2$ of $2$-adic numbers. In this case, obviously, the residue field $k$ consists of two elements. For an arbitrary matrix $a$, we shall always denote by $a_{ij}$ the element standing at the intersection of the $i$-th row and the $j$-th column. By $d_i(\xi)$ we denote the diagonal matrix $e + (\xi - 1)e_{ii}$, $N(e) = 1$.

Following [5], the pair $(x, y)$ of elements of the field $K$ will be called compatible, if $N(x) + N(y) = 1$. For a compatible pair $(x, y)$, we denote the unitary matrix of the form

$$\begin{pmatrix} x & y \\ \bar{y} & \bar{x} \end{pmatrix}$$

by $T(x, y)$. For a pair of distinct indices $i$ and $j$ ($1 \leq i, j \leq n$), the matrix $T_{ij}(x, y)$ we denote the image of the matrix $T(x, y)$ under the standard imbedding of the group $U(2, K)$ in $U(n, K)$.

It is shown in [5] that for some classes of quadratic extensions $K/k$ (called there $\mathbb{Q}$-extensions) the unitary group $U(n, K)$ is generated by the subgroup of diagonal unitary matrices $DU(n, K)$ and all matrices $T_{ij}(x, y)$. Now we shall show that the analogous assertion