BOUNDARY-LAYER SEPARATION ON CONICAL BODIES

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Some characteristics of the variation in the linear dimensions of the flow separation zones on conical bodies with expanding conical skirts and of variation of the pressure within these zones as a function of variation of the Mach number, Reynolds number, and intensity of the disturbance that causes the boundary layer separation are examined. Experiments were conducted in laminar, transitional, and turbulent flows in flow separation regions.

The interaction of viscous and nearly inviscid flows is quite common. This phenomenon occurs in flow past a concave corner, when a compression shock impinges on a boundary layer, and in many other cases. The characteristics of this phenomenon in flow about two-dimensional bodies have been investigated experimentally in [1,2] and other studies. Attempts have been made to analyze the interaction of compression shocks with the boundary layer theoretically. In "free" separated flows, when the points of separation and reattachment of the boundary layer are not fixed (for example, on a flat plate with a long wedge attached to it), theoretical studies are usually made within the framework of the boundary layer theory with use of the approximate integral methods [3,4]. In this article we examine some results from studies of free separated flows on conical bodies with conical skirts in laminar, transitional, and turbulent flows (Fig. 1).

NOTATION

- $x$: distance along axis
- $y$: distance from axis to surface of body
- $s$: distance along wetted surface
- $\delta$: deviation of skirt surface generator from forward cone generator
- $\delta_v$: boundary-layer thickness
- $\delta^*_{v}$: displacement thickness
- $\delta^{**}_{v}$: momentum thickness
- $u$: velocity
- $M$: Mach number
- $Re$: Reynolds number
- $\rho$: density
- $p$: pressure
- $r$: friction stress
- $C_{p}$: pressure coefficient
- $C_{f}$: friction coefficient
- $t$: temperature
- $t_{0}$: stagnation temperature

Subscripts:

- $0$: beginning of interaction region
- $1$: boundary-layer separation
- $2$: cone generatrix break point
- $3$: reattachment point of separated layer
- $4$: end of interaction region
- $w$: flow parameters at the wall
- $e$: parameters of outer inviscid flow.

1. For free separated flows the positions of the boundary layer separation and reattachment points are not initially known and are determined only by the interaction conditions. To calculate the flow of a laminar boundary layer in the attached and separated flow regions we will use the Cohen-Reshotko method [5], and to describe the flow of the inviscid gas we will use the equations for Prandtl-Meyer flow.

We obtain the following system of equations:

\begin{align*}
\frac{d}{ds} \left( \rho u\delta^{**}_{v} y \right) + \delta^{**}_{v} \frac{dp}{ds} &= y t_{w}, \\
\frac{d\theta}{ds} &= \frac{YM^2 - 1}{\rho Y M^2} \frac{dp}{ds}, \\
\frac{dY}{ds} &= \frac{dx}{ds}, \\
\frac{dp}{ds} &= \frac{\mu_{w} U}{\delta^{**}_{v}} \left( \frac{t_{e}}{t_{w}} \right) \left( \frac{t_{e}}{t_{0}} \right).
\end{align*}

We assume that the flow deflection angles are small and that the displacement thickness edge is described approximately by the equation $Y = y(S) + \delta^{**}_{v}$.

It is also assumed that the flow is described by a single-parameter family of profiles and that $n$ will be only a function of $H = \delta^{v}_{v}/\delta^{**}_{v}$.

In this system (1.1) describes the boundary layer flow, (1.2) describes the inviscid gas flow, (1.3) defines the relation between the deflection of the outer flow and the change of displacement thickness, and (1.4) establishes the connection between the local pressure gradient and the form parameter $n$.

Equation (1.1) was solved by the Cohen-Reshotko method [5], and as the single-parameter family of profiles we used the family of solutions of the Falkner-Skan equation for both the separation-free and the separated flow regions. We now specify the boundary conditions. We assume that $\delta^{v}_{v}$, $\delta^{**}_{v}$, $\theta_{0}$, $p_{0}$, and $M_{0}$ are known at the initial section $x = x_{0}$.

At the end of the Interaction region, at section $x_{4}$, the solution must satisfy the following conditions: the outer flow must be parallel to the surface of the conical skirt, i.e., the condition $\theta_{4} = \beta$ must be satisfied, and the flow conditions at the skirt surface must be the
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same as on the sharp cone. For this we require that $\psi_s' = 0$. The boundary conditions at the end of the interaction zone for each given value of the skirt generatrix deflection angle $\beta$ were satisfied by selecting the position of the initial section $x_0$ by the "shooting" method. In [6] a similar method was used to calculate the flow in the interaction region on a flat plate with a wedge mounted on it. It is interesting to determine the dependence of the pressure in the separated flow region and the length of this region on the numbers $M$ and $R$ and on the pressure coefficient $C_p$ at the end of the interaction region. It was shown in [2] that the pressure coefficient $C_p$ in the "plateau" region must be proportional to $[(M^2 - 1)R_1]^{-1/4}$.

Analysis of the experimental results of [1, 2] and others shows that within $\pm 15\%$ the pressure coefficient $C_p$ in the plateau region on a flat plate (at the wetted surface break point) is described by the relation

$$C_p = 1.67[(M^2 - 1)R_1]^{1/4}. \quad (1.5)$$

However, in the case of laminar boundary-layer separation on a cone, the pressure coefficient in the plateau region is higher by a factor of $\sqrt{3}$ than that on the flat plate. In fact, if the separation criterion for the laminar boundary layer on the cone and on the flat plate has the same value,

$$\Gamma \sim 3^2 \psi^2 / \mu U \sim (d C_p / dx) \sqrt{F N}, \quad (1.6)$$

Here $z$ is taken equal to $\delta_s^*$. The pressure coefficient change in the outer flow is proportional to

$$\Delta C_p \sim (d \delta^* / dx) / \gamma M^2 - 1 \approx (\delta_s^* / x) / \gamma M^2 - 1.$$ \text{Therefore,}

$$C_p \sim (\Gamma / \gamma (M^2 - 1) R_1^{1/2}).$$

But on the cone $\delta_s^*$ is smaller by $\sqrt{3}$ times than on the flat plate and, consequently, on the cone in the plateau region

$$C_p \sim (3^2 \Gamma / \gamma (M^2 - 1) R_1^{1/2}). \quad (1.7)$$

Figure 2 shows a comparison of the experimental and theoretical values of $C_p$ on the cone with half-angle $10^\circ$ and $M = 5.0$ on the cone surface ahead of the separation point and the corresponding flat-plate data. Curve 1 is the variation of the experimentally measured values of $C_p$ on the flat plate as a function of the number $R$ ahead of the separation point (the points show the experimental values of $C_p$ on the cone); curve 2 is the theoretical relation for $C_p$ on the cone; and curve 3 corresponds to the data on the coefficient $C_p$ for the flat plate, multiplied by $\sqrt{3}$. The theoretical and experimental data agree quite well.

Next we evaluate the nature of the dependence of the linear dimensions of the laminar separated flow region on the freestream parameters and the conical skirt deflection angle. If the skirt dimensions are large and the separation is free, the separated flow zone length $l$ from the point of boundary layer separation to the wetted body generatrix break point is directly proportional to the boundary-layer thickness or the cone length ahead of the separation point, since there are no other characteristic parameters.

We attempt to explain the nature of the dependence of $l/\delta_s^*$ on $\beta$ for completely laminar flow. We know that the pressure in the interaction region at first increases quite slowly and then more rapidly and that the pressure gradient reaches a maximum near the separation point. Further downstream the pressure gradient decreases to nearly zero ("pressure plateau"). Then the pressure again begins to increase, and the pressure gradient reaches a new maximum in the region of reattachment of the separated layer, after which the pressure gradient again decreases. The maximum value of the pressure gradient at the separation point is defined by a separation criterion of the following form:

$$\Gamma = (d C_p / dx) \sqrt{F N} \quad (1.7)$$

If we assume that the flow in the boundary layer is described approximately by a single-parameter family of profiles, the reattachment of the separated layer is also defined by a criterion like (1.7).

Separation criterion (1.7) thus limits the maximum value of the pressure in the interaction region. Therefore, the magnitude of the increase in the skirt surface deflection angle and, consequently, the pressure coefficient at the end of the interaction region lead to an increase in the dimensions of the separated flow region. For a rough estimate of the separated zone length we apply criterion (1.7) to the entire interaction region. Then the pressure coefficient increment $\Delta C_p \sim C_p$, since $C_p$ varies from 0 to $C_p$ in the interaction region. Since this change takes place over the entire length of the interaction region, we assume that $\Delta x \sim l$, the length of the interaction region. When the separated flow region is not large and the thickness of the separated zone exceeds only slightly the thickness of the attached boundary layer, we can take as the characteristic transverse dimension $z$ the displacement thickness $\delta_s^*$ ahead of the boundary layer separation point. Then, using (1.7), we obtain

$$l/\delta_s^* \sim C_p \sqrt{R_0}. \quad (1.8)$$