TABLE 1

<table>
<thead>
<tr>
<th>ϵ</th>
<th>0.1</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>ω</td>
<td>0.44</td>
<td>0.49</td>
<td>0.60</td>
<td>0.77</td>
<td>1.00</td>
<td>1.25</td>
<td>1.50</td>
</tr>
<tr>
<td>ρ(ε)</td>
<td>0.06</td>
<td>0.28</td>
<td>0.52</td>
<td>0.71</td>
<td>0.85</td>
<td>0.92</td>
<td>0.97</td>
</tr>
<tr>
<td>ρ(ω,ε)</td>
<td>0.16</td>
<td>0.54</td>
<td>0.92</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

In particular, for $N(m, \sigma^2)$, we have

$$
\hat{\rho}_2(\epsilon) = \frac{n-3}{2\pi(n-2)^{1/2}} \int_{\mathbb{R}^2} h(x,y) \left[ \psi(x, y) \right] \left[ \psi(x, y) \right]^{(n-2)/2} dx dy,
$$

where $\psi(x, y) = 1 - \left( (x^2 + y^2 - (x - y)^2) / (n-2) \right)$, $s$ is the sample variance, and $\Phi$ is the Laplace function. The estimator $f(x, y, \theta)$ for the multivariate normal distribution was obtained in [2].

**LITERATURE CITED**


**COMPUTING THE DISTRIBUTION FUNCTION OF THE RATIO OF QUADRATIC FORMS IN NORMAL VARIABLES**

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UDC 519.2

Formulas are given for computing the ratio of quadratic forms in normal variables. The doubly noncentral $F$-distribution function is computed.

Consider the distribution function $F(x)$ of the ratio

$$
F(x) = \rho \left\{ \sum_{i=1}^{n} \alpha_i \chi^2_{n_i}, \delta_{i}, \epsilon_{i} \right\} / \sum_{j=1}^{m} \alpha_{2j} \chi^2_{n_{2j}}, \delta_{2j}, \epsilon_{2j}
$$

of linear combinations of independent random variables $\chi^2_{n_k}, \delta_{k}, \epsilon_{k}$, which for each $k$ and $l$ are distributed with $n_k$ degrees of freedom and noncentrality parameter $\delta_k, \epsilon_k$. All $\alpha_{kl} > 0$. The random variable (1) is obtained by taking the ratio of two quadratic forms in normal variables. The distribution function $F(x)$ may be represented in the form

$$
F(x) = \rho \left\{ \sum_{i=1}^{n} \alpha_i \chi^2_{n_i}, \delta_{i}, \epsilon_{i} - \sum_{j=1}^{m} \alpha_{2j} \chi^2_{n_{2j}}, \delta_{2j}, \epsilon_{2j} \right\}
$$

Our problem is thus to find the values of the distribution function of the linear combination
\[ \xi = \sum_{i=1}^{n} \alpha_{ti} \chi_{n_{i1}}^{\delta_{t1}} \delta_{t1} + \sum_{j=1}^{m} \beta_{tj} \chi_{n_{j2}}^{\delta_{tj}} \delta_{tj} \]
of \( \chi^2 \)-distributed random variables with coefficients of different signs.

Two basic approaches are available for the computation of the distribution function of such random variables: (a) by series expansion of the distribution function [4] and (b) by numerical integration of the inversion formula [1, 5]. Approach (b) is more suitable for numerical implementation and is therefore used in this paper.

The characteristic function of \( \xi \) is
\[ \varphi(t) = \varphi(t, x) = \prod_{j=1}^{n} \exp \left\{ \frac{i t \delta_{tj}}{1 - 2 i \alpha_{tj} t} \right\} \left( 1 - 2 i \alpha_{tj} \varphi \right)^{-n_{j1}/2} \times \prod_{j=1}^{m} \exp \left\{ \frac{i t \delta_{tj}}{1 + 2 i \alpha_{tj} \varphi} \right\} \left( 1 + 2 i \alpha_{tj} \varphi \right)^{-n_{j2}/2}. \]

The inversion formula for the characteristic function is [2]
\[ G(y) = \frac{1}{2} = \frac{1}{\pi} \int_{0}^{\infty} \frac{1}{4} \Im \left[ \varphi(t, x) \cdot \exp \{-i t y\} \right] \, dt. \]

For our distribution function \( F(x) \), we thus have the formula
\[ F(x) = G(x) = \frac{1}{2} - \frac{1}{\pi} \int_{0}^{\infty} \frac{1}{4} \Im \left[ \varphi(t, x) \right] \, dt. \]

As in [1], each factor \( \varphi(t, x) \) for the characteristic function of a quadratic form is representable in the form \( re^{i\theta} \) as follows:

\[ \exp \left\{ \frac{i t \delta_{tj}}{1 - 2 i \alpha_{tj} t} \right\} = \exp \left\{ - \frac{2 i \alpha_{tj}}{1 + 4 \alpha_{tj} t} \right\} \exp \left\{ \frac{i t \delta_{tj}}{1 + 4 \alpha_{tj} t} \right\}; \]
\[ \exp \left\{ \frac{i t \delta_{tj}}{1 + 2 i \alpha_{tj} \varphi} \right\} = \exp \left\{ - \frac{2 i \alpha_{tj}}{1 + 4 \alpha_{tj} \varphi} \right\} \exp \left\{ i t \delta_{tj} \right\}; \]
\[ (1 - 2 i \alpha_{tj} \varphi)^{-n_{j1}/2} = (1 + 4 \alpha_{tj} \varphi)^{-n_{j1}/4} \exp \left\{ - \frac{n_{j1}}{2} \arctan \left( 2 \alpha_{tj} \varphi \right) \right\}; \]
\[ (1 + 2 i \alpha_{tj} \varphi)^{-n_{j2}/2} = (1 + x^2 \alpha_{tj} \varphi)^{-n_{j2}/4} \exp \left\{ - \frac{n_{j2}}{2} \arctan \left( 2 \alpha_{tj} \varphi \right) \right\}. \]

Calculating the product of the amplitudes \( r \) and summing the phases \( \theta \), we obtain after replacing \( 2t \) with \( t \)
\[ F(x) = \frac{1}{2} - \frac{1}{\pi} \int_{0}^{\infty} \frac{1}{4} \exp \left\{ - \sum_{i=1}^{n} \alpha_{t1} \delta_{t1} \frac{x_{i}}{1 + \alpha_{t1} x_{i}^{2} + \frac{1}{4} (1 + x^{2} \alpha_{t1} \delta_{t1})} \right\} \times \prod_{i=1}^{n} \left( 1 + \alpha_{t1} x_{i}^{2} \right)^{-n_{i1}/4} \times \prod_{i=1}^{m} \left( 1 + x^{2} \alpha_{t1} \delta_{t1} \frac{x_{i}}{1 + \alpha_{t1} x_{i}^{2} + \frac{1}{4} (1 + x^{2} \alpha_{t1} \delta_{t1})} \right) \times \sin \left( \sum_{i=1}^{n} \frac{n_{i1}}{2} \arctan \left( \alpha_{t1} \delta_{t1} \frac{x_{i}}{1 + \alpha_{t1} x_{i}^{2}} \right) + \frac{\alpha_{t1} \delta_{t1} x_{i}}{2 (1 + \alpha_{t1} x_{i}^{2})} \right) \right] \times \]