\[ + \left| \lambda \right| \int_{a}^{b} \left| K(x_{0}, s) - K(z_{m}, s) \right| \| \Psi(s, A_{m}) \| \, dt + \left| L[\Psi(z_{m}, A_{m})] \right| \leqslant \left( \sum_{i=0}^{k} |a_{i}| \right) H_{i} + H + \left| \lambda \right| K_{1}N(b - a) \left| x_{0} - z_{m} \right| + \rho_{m}, \]

where

\[ \left| \psi_{i}(x_{0}) - \psi_{i}(z_{m}) \right| \leqslant H_{i} \left| x_{0} - z_{m} \right|, \quad i = 0, 1, \ldots, k, \]

\[ \left| f(x_{0}) - f(z_{m}) \right| \leqslant H \left| x_{0} - z_{m} \right|, \]

\[ \left| K(x_{0}, s) - K(z_{m}, s) \right| \leqslant K_{1} \left| x_{0} - z_{m} \right|. \]

From the last expression it follows that \( \rho \leqslant \lim_{m \to \infty} \rho_{m} \). However, for all \( m \), \( \rho_{m} \leq \rho \). Thus, \( \lim_{m \to \infty} \rho_{m} = \rho \).

The sequence of minimizing generalized polynomials thus converges on point sets to the solution of the integral equation (1).

**LITERATURE CITED**


**A BOUNDARY-VALUE PROBLEM FOR SYSTEMS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS**

V. V. Marinets

We investigate a boundary-value problem for systems of nonlinear partial differential equations and construct a modified two-sided method for approximate integration of this problem. We assume that the right-hand side of the system is a continuous function with bounded first partial derivatives in the given domain.

Consider the system of differential equations

\[ D_{k}^{2}U(x, y) = F(x, y, U(x, y), D^{0,0}U(x, y), \ldots, D^{k-1,0}U(x, y)), \]

\[ \ldots, D_{k}^{k}U(x, y) = F[U(x, y)], \]

\[ k = (k_{1}, k_{2}), \quad k_{1} = 0, 1, 2; \quad k_{2} = 0, 1; \quad |k| \leqslant 2 \]

with the boundary conditions

\[ U(x, 0) = T(x), \quad 0 \leqslant x \leqslant a, \quad U(a, y) = \Phi(y), \]

\[ D_{k}^{0,0}U(a, y) = \Psi(y), \quad 0 \leqslant y \leqslant b, \]

\[ (1) \]

where
\[\text{den} \; D^kU: \overline{B}_0 \to \overline{B}_{\beta_0} \subset \mathbb{R}^2, \; \overline{B}_0 = \{(x, y)\mid 0 \leq x \leq a, \; 0 \leq y \leq b\}, \; F: \overline{D} \to \mathbb{R}^2, \; \overline{D} = \overline{B}_0 \times \overline{B}_0 \times \cdots \times \overline{B}_0 \subset \mathbb{R}^{2n+2}, \; D^kU(x, y) = (D^kU_1(x, y)), \; F[U(x, y)] = (f_1[U(x, y)]), \; T(x) = (\xi(x)), \; \Phi(y) = (\psi_1(y)), \; \Psi(y) = (\psi_i(y)), \; i = 1, n\]
are column vectors.

In what follows, we assume that \(T(x) \in C^2[0, a], \Phi(y), \Psi(y) \in C^1[0, b], F[U(x, y)] \in C_1(\overline{D})\) - the class of continuous vector functions with bounded first partial derivatives in \(\overline{D}\) with respect to all arguments starting with the third.

Existence and uniqueness theorem for problem (1), (2) is proved in [1] for the case of a scalar linear equation. The purpose of the present study is to generalize the results of [1] to nonlinear systems and to construct approximate integration methods for these systems.

We rewrite the boundary-value problem (1), (2) in equivalent integral form
\[U(x, y) = \Omega(x, y) + \int_0^s s \int_0^x (\xi - x) F[U(\xi, t)]d\xi dt \equiv \Omega(x, y) + HF[U(\xi, t)], \tag{3}\]
where
\[\Omega(x, y) = T(x) + \Phi(y) - \Phi(0) + (a - x)(\Psi(0) - \Psi(y)), \]
\[\Omega(x, y) = (\omega_i(x, y)), \]
and represent the right-hand side of system (1) in the form
\[F[U(x, y)] = F[U^+(x, y), U^-(x, y)],\]
where
\[D_j^kF = \frac{\partial F}{\partial D^kU^i}(x, y) \geq (\geq) 0, \]
\[D_j^kF = \frac{\partial F}{\partial D^kU^i}(x, y) \leq (\leq) 0, \; i = 1, n; \]
\[k_1 = 0, 2 (k_1 = 1), \; k_2 = 0, 1. \tag{4}\]

Assume that vector functions \(Z_0(x, y), V_0(x, y) \in C^{(2,1)}(\overline{B}_0)\) exist in \(\overline{D}\) that satisfy conditions (2) and the inequalities
\[D^2Z_0(x, y) - D^2V_0(x, y) \equiv D^2W_0(x, y) \geq (\geq) 0, \]
\[k_1 = 0, 2 (k_1 = 1), \; k_2 = 0, 1, \]
\[D^{(2,1)}Z_0(x, y) - F[Z_0(x, y), V_0(x, y)] \geq 0, \]
\[D^{(2,1)}V_0(x, y) - F[V_0(x, y), Z_0(x, y)] \leq 0. \tag{5}\]

The set of these vector functions is nonempty.

Denote
\[\alpha_p(x, y) = D^{(2,1)}Z_p(x, y) - F[Z_p(x, y), V_p(x, y)], \]
\[\beta_p(x, y) = D^{(2,1)}V_p(x, y) - F[V_p(x, y), Z_p(x, y)], \]
\[F_p = (f_p^i), \; F_p = (f_i,p), \; i = 1, n, \]
\[f_p^i = f_1[z_{i-1,p+1}, \ldots, z_{i,p+1}, z_{i,p}, \ldots, z_{n,p}, v_{i,p+1}, \ldots, v_{i-1,p+1}, v_{i-1,p}, \ldots, v_{n,p}, z_{i+1,p+1}, \ldots, z_{n+1,p+1}, z_{n,p}], \]
\[f_i,p = f_1[v_{i,p+1}, \ldots, v_{i-1,p+1}, v_{i-1,p}, \ldots, v_{n,p}, z_{i+1,p+1}, \ldots, z_{i-1,p+1}, z_{i,p}, \ldots, z_{n,p}]. \tag{6}\]