 Canonical variables (γ-variables) are constructed for a quasiperiodic classical XXZ-chain (of the discrete Landau-Lifshits equation with monoaxial anisotropy). The approach, previously suggested by the author for the XYZ-model (J. Sov. Math., 46, No. 1 (1989)), is used.

The present paper continues the author’s series of papers [1, 2, 3], devoted to the problem of determining the spectrum of the integrals of motion (in the quantum case) or the construction of action-angle variables (in the classical case) for models which are integrable in the framework of the r-matrix method [4]. In [3] the construction of the separation of variables suggested in [1] and [2] for a Goryachev-Chaplygin top and a periodic Toda chain, respectively, was generalized to the case of a classical periodic XYZ-chain. In the present paper we give a summary of the analogous results for a classical quasiperiodic XXZ-chain.

We shall not explain here the formulation of the problem of separation of variables and the origin of the method proposed, but only their value for quantization, since all this is described in detail in [3]. We note only that although the XXZ-model is a degenerate case of an XYZ-model, its study is of independent interest, since among the continuous limits of

XXZ-magnetics there are models of such importance in applications as the Landau-Lifshits equation with monoaxial anisotropy and the sine-Gordon equation. Moreover, under degenera-
tion XYZ \rightarrow XXZ one can, on the one hand, essentially simplify the complicated formulas of [3], and on the other hand, include quasi-periodic boundary conditions of more general form than for the XYZ-case. Information about the limit passage XYZ \rightarrow XXZ is only given for comple-
teness of the exposition. As in [3], we only consider the complex case here and the problems of real reduction are not discussed.

In point 1 the basic definitions and notation are collected. The results of [3] are reproduced in points 2 and 3 for XXZ-magnetics. Namely, the so-called natural variables of [3] are constructed in point 2, and the expression for the monodromy matrix in terms of the natural variables is given in point 3. The new features of XXZ-magnetics compared with the XYZ-case are discussed in point 4; and the connection of the formulas of [3] and of the present paper are outlined in point 5.

1. XXZ-magnetics corresponds to the following solution of the classical Yang-Baxter equation [4]

$$ \tau (u) = - \frac{i}{2} \sum_{\ell = 1}^{3} \tilde{w}_\ell(u) \hat{e}_\ell \hat{e}_\ell, $$

(1)

where

$$ \tilde{w}_\ell(u) = \tilde{w}_\ell(u) - 1/\sinh(u), \quad \tilde{w}_\ell(u) - e^{iu}. $$

With the r-matrix (1) there is connected in the standard way [4] the Poisson square bracket

$$ \{T(u_1), T(u_2)\} = \tau_T(u_1 - u_2), \quad T(u) = e^{iu}, \quad \frac{d}{du} T(u) = \tau_T(u). $$

(2)

The simplest orbits (symplectic leaves) of the bracket (2) are connected zero-dimensional orbits, that is, constant matrices which satisfy the condition

$$ \{\tau(u), \Phi \otimes \Phi\} = 0 \quad \forall u. $$

(3)

For the r-matrix (1), \( \Phi \) can assume the values

$$ \Phi = e^{i\sigma_3}, $$

(4a)

or

$$ \Phi = \sigma_j e^{i\sigma_3}, $$

(4b)

where \( \sigma_j \in \mathbb{C} \) is a continuous parameter. We note that for the XYZ-model \( \Phi \) can assume only the values \( \Phi = 1, \sigma_1, \sigma_2, \sigma_3 \) (up to a constant factor).

The next orbits of the bracket (2) in complexity are the L-operators, that is, matrix-valued functions \( L(u) \) with simple dependence on the parameter \( u \). We shall consider L-opera-
tors of the form

$$ L(u) = \begin{pmatrix} S^\alpha \sinh(u) + S^\beta \cosh(u) & S^\gamma \\ S^\gamma & S^\alpha \sinh(u) - S^\beta \cosh(u) \end{pmatrix}. $$

(5)

Substituting \( T(u) = L(u) \) in (2), we get the Poisson brackets for the variables \( S^\alpha (\alpha = 0, 3, \pm) \)

$$ \{ S^\alpha, S^\beta \} = 0, \quad \{ S^\alpha, S^\pm \} = i S^\beta S^\pm, \quad \{ S^+, S^- \} = 2i S^\alpha S^\beta, \quad \{ S^-, S^+ \} = i S^\beta S^\pm. $$

(6)

We shall assume that the constraint

$$ (S^\alpha)^2 + S^+ S^- = \sinh^2, \quad (S^\alpha)^2 -(S^\beta)^2 = 1, $$

or

$$ \det L(u) = \sinh^2(u - l) - \sinh(u + l) \sinh(l) $$

(7)